Optimal Shunt Compensators at Nonlinear Busbars

by

Bawah, Abdullah Umar

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

ELECTRICAL ENGINEERING

July, 1992

Order Number 1354067

Optimal shunt compensators at nonlinear busbars

Bawah, Abdullah Umar, M.S.

King Fahd University of Petroleum and Minerals (Saudi Arabia), 1992

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

OPTIMAL SHUNT COMPENSATORS AT NONLINEAR BUSBARS

By

Bawah, Abdullah Umar

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment Of The

Requirement For The Degree Of

Master Of Science

In

Electrical Engineering

July 1992

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN 31261,SAUDI ARABIA.

COLLEGE OF GRADUATE STUDIES

This thesis, written by **Bawah , Abdullah Umar** under the direction of his Thesis Advisors and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Thesis Committee

DR..El-Amin, M. Ibrahim
Chairman

Dr. Al-Shehri, M. Abdallah
Member

Dr. Opoku,George
Member

Dr. Saleh Duffuaa
Member

Chairman
Department of Electrical Engineering

Dr. Ala H. Al-Rabeh
Dean
College of Graduate Studies
Date : 6 7 02

DEDICATION

THIS THESIS IS DEDICATED TO MY MOTHER

AND

MY DAUGHTER FAAIZA

ACKNOWLEDGEMENTS

Acknowledgement is due to the King Fahd University of Petroleum and Minerals (KFUPM) for providing the author with employment and other academic facilities to carry out this work. Thanks to all the persons who helped and inspired the carrying out of this work. The first of these persons is Dr. Ei-Amin. M. Ibrahim of the Electrical Engineering department of KFUPM. who played the role of the chairman of this thesis committee. He helped in providing numerous suggestions as to how this work should be carried out. Acknowledgment is also due to the other members of the committee Dr. Al-Shehri, M. Abdallah and Dr. Opoku George both of the Electrical engineering department of the KFUPM and Dr. Saleh Duffuaa of System engineering department of the KFUPM. The later helped and guided the author through the best optimaization techniques best suited for this work. The assistance provided by the Electrical engineering department as well as the Professors of the department was a valuable contribution to this work. Finally, all friends who provided the author with the moral support to go through this work are acknowledged for their contributions.

THESIS ABSTRACT (English)

FULL NAME OF STUDENT : Umar Abdullah Bawah

TITLE OF STUDY : Master of Science

MAJOR FIELD : Electrical Engineering

DATE OF DEGREE : July 1992

A model for determining an optimal shunt capacitor value at nonsinusoidal busbars to meet three basic criteria - maximising the power factor, minimising the line losses. and maximising the transmission efficiency is developed . The choice of the capacitor value is constrained by the values that may cause resonance. This model employs the Golden Section Search algorithm as a solution procedure. It also made use of the penalty function to combine all the three criteria in one objective function to provide a single solution meeting all the three criteria. The advantage of this method over other conventional approaches is that. it guarantees convergence to the optimal solution. The solution approach is further extended to include nonlinear loads operating under nonsinusoidsal conditions. The model for the nonlinear load is a two variable problem in L (inductor) and C (capacitor). The Direct Search Polytope algorithm for multivariable function is used as the solution procedure . The solution of this model yields an optimal shunt LC compensator for the nonlinear load. However, since there are limitations on the practical values of shunt capacitor, a discretising approach making use of standard shunt capacitor values is employed to guarantee a solution that can be implemented practically.

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Dhahran, Saudi Arabia

Date : July 1992

الخلاصة

اسم الطالب : عمر باوا عبدالله .

عنوان الرسالة : التعويض الموازي الأمثل في الأحمال الغير خطية

التخصص : هندسة كهربائية .

التاريخ : يوليو ١٩٩٢م .

لقد تم إنجاز نموذج لحساب قيم المكثفات الموازية عن النقاط غير المترددة وذلك على أساس تحقيق ثلاثة مقاييس وهي الوصول إلى أعلى قيمة لمعامل القوى ثم التقليل من خسارة القوة في الخطوط ثم تحقيق أعلى أداء في إرسال القوى . إن إختيار قيمة المكثف مشروط بتجنب حالة الطنين . إن هذا النموذج يعتمد على طريقة فولدن في البحث كأسلوب في الحل كما أن هذا النموذج يستعمل دالة معامل الجزاء حتى يدمج الثلاث مقاييس وذلك في الدالة المستهدفة من أجل الحصول على حل أوحد يتماشى مع الثلاث مقاييس المذكورة .

إن ميزة هذه الطريقة مقارنة بالطرق التقليدية الأخرى هي في ضمان الإقتراب نحو الحل الأمثل .

لقد تم التوسع في طريقة الحل لكي تشمل الأحمال الغير خطية والتي تكون تحت ظروف غير تسدديه .

إن نموذج الأحمال الغير خطية هي مشكلة ذات متغيرين في الحالتين L وفي المكثف C . لقد استعملت طريقة البحث المباشر للدالة ذات المتغيرات المتعددة كطريقة للحل .

إن الوصول إلى حل لهذا النموذج يؤدي إلى إختيار أمثل لتركيبة LC كموازي للتعويض في حالة الأحمال الغير خطية . بما أن هناك حدود لقيم المكثف الموازي فإن إتباع طريقة التقطيع بإستعمال المكثف المتوازن وتنفيذها يضمن حل ممكن تطبيقه عملياً .

درجة الماجستير في العلوم الهندسية

جامعة الملك فهد للبترول والمعادن

الظهران ، المملكة العربية السعودية

يوليو ١٩٩٢م

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

The subject of harmonics has been deemed so important that there are international conferences dedicated to the subject, working groups and committees in international engineering societies which deal only with harmonics, and several dedicated books on the topic. Harmonics in the terminology of electric power systems, represent the deviation from the sinusoidal nature of the voltage or current to that of a distorted function consisting of the fundamental frequency of operation plus additional frequency components termed the harmonic frequencies. Such a situation is described as being NONSINUSOIDAL. The presence of harmonics have some undesirable effect on power equipment. The need to identify the source of harmonics and how to correct their side-effects in order to deliver quality power to customers , has attracted the attention of electric power engineers for a long time now.

The source of harmonics in power systems has largely been attributed to the operation of some types of load or power equipment [1]. Loads with harmonic impact on the power system include a wide varying load natures but not limited to the following :

(i) Converters : -

These are electrical devices capable of changing electrical energy from one form to another - usually from AC to DC or vice - versa, or from one frequency to another . Table 1.1 gives us the types of converters in use , their practical applications and power ratings [2]. Converters produce nonsinusoidal voltage and current waveforms. The harmonics levels vary from one converter configuration to another. Electric utilities have therefore developed standards and guidelines for the installation of converters to prevent them from leading to waveform distortions .

(ii) Rotating AC machines :

Most of these machines are designed in a way that the pole faces can cancel low order harmonics from the supply. This depends on the machine type and cancellation is through the 11 th harmonic.

(iii) Fluorescent lighting :

As a result of its high efficiency. the fluorescent lamp has become the lamp of choice when very large areas must be lit efficiently. The impact of fluorescent lighting on AC distribution systems is mainly that of high third harmonic content. This third harmonic content appears as zero sequence in the neutral of three phase systems. For substations at which the connected fluorescent load exceeds 10 % of the total KVA rating. network resonances at or near the the third harmonic may give unacceptably high third harmonic voltage.

(iv) Glow discharge lighting :

Xenon, Neon, Krypton, Mercury vapor . pressurized Sodium vapour, and others are lamps of this group used mainly for special purposes. As they are special purpose lamps , they are rarely used and very expensive. The supply current of all these types is similar to that observed in fluorescent lighting.

(v) Over excited transformers :

Over excited transformers will cause harmonic voltages in the network. The extent of the harmonic impact increases with the degree of over excitation.

(vi) Transfomer magnetizing current :

An additional source of harmonics from transformers is the nonsinusoidal wave shape of the magnetizing current. Transformer magnetizing current depend on the supply voltage amplitude.

(vii) Arc Furnaces :

Arc furnaces are industrial devices used for melting and refining metals, such as the process of using iron to produce steel . They are therefore used to apply bulk energy through the application of high current . There are both AC and DC arc furnaces used in the production of steel . The process involves three stages. The first is the lowering of the electrodes of the supply into a vessel charge (i.e., into the scrap steel in the vessel) in order to initiate the electric arc. The second stage is the melting stage in which the arc applies heat to the surface of the vessel charge and the current paths in the vessel result in I^2R heating. In this stage, the currents may be quite high and the waveshape of the current may be quit erratic since the paths for the currents are rapidly changing . The third stage is the refining stage in which the main heating mechanism is that of I^2R heating due to current in the charge. The arc furnances imposes harmonic problems on the power system depending on their size and location.

Table 1.1 Converters used in power systems.

Type	Application	Typical ratings
Rectifiers		
Single phase half wave and full wave	• Electronic devices • Television and communications receivers • Small computers • Battery chargers	0 - 30 kW
Three-phase, six-pulse	• Commercial DC sources • Electroplating • Battery chargers • Ultrasonic heaters • DC motors • Mainframe computers • Radio transmitters	20 - 1000 kW
Three-phase, twelve-pulse	• DC motors • Industrial DC sources • Transportation systems • DC arc furnaces • Smelters • Electrolysis cells • SVC's • HVDC transmission	Above 1 MW
High phase order	• Smelters • DC arc furnaces • Electrolysis cells	Above 50 MW
Inverters		
Three-phase, six-pulse	• Small solar photovoltaic panels • UPS's	Up to 1 MW
Three-phase, twelve-pulse	• Solar photovoltaic panels • UPS's • HVDC systems	Above 1 MW

The nonlinear voltage - current relationship of these devices result in harmonic currents which propagate through the system and produce potentialy dangerous harmonic currents [1]. These type of loads are therefore described as nonlinear loads and are usually represented by ideal current harmonic sources at their characteristic frequencies. Amplification of harmonic currents or voltages under resonance condition can have detrimental effects on other elements of the system. Among these effects are the degrading of the power factor of the load (to be defined later), increasing the transmission line losses and hence decreasing the transmission network efficiency. Harmonic propagation in a system largely depend on the network configuration and network component parameter values. Among these parameters is the shunt capacitor usually installed at consumer premises to improve the power factor. Specifically, capacitors significantly affect the propagation of distribution system harmonics.They are used in large numbers in distribution systems for loss reduction and voltage control, thus increasing the combinations of circuit parameters that can produce series resonance at or near the frequency of low-order characteristic harmonic.

The 20-th century has witnessed an ever increasing use of these type of devices at both domestic and industrial consumer's premises. Severe problems are therefore imposed on the electric utilities such as the harmonic currents injection in the utility customer's supply feeder as described earlier on [1]. However, the distortion in the A.C. bus voltage has different effect on different loads . For example , an arc furnace load will be relatively insensitive to distortion in the A.C. bus voltage , on the other hand , the operator of a computer centre cares much about the slightest distortion in the A.C. bus voltage as this can lead to false timing signals , interference in digital data lines (by induction) , and improper grounding [2]. To satisfy the service condition of these varying nature of load . it is therefore not out of question for an electric power engineer to spend substantial time and energy to address the issue of reactive shunt compensation at nonlinear busbars.

The main approach to the problem of degrading power factor is to completely or partially compensate for the inductive reactive power of the load. Conventional methods apply shunt capacitors for the purpose of generating capacitive reactive power which compensates the inductive reactive power required by the load. This yields an increased power factor (the ratio of active power to apparent power) at the load and reduces the line current from the source as a result of which the power losses are minimised and the transmission efficiency is maximised. However. it is worth mentioning that these conventional methods have two different approaches: (1) by considering that the supply is devoid of harmonics. that is . it is a pure sinusoid and also the loads are considered to be linear. These assumptions make it possible to determine a capacitor value which will improve the power factor(PF) to any desirable value including the optimum- unity power factor. Erroneous results are obtained in this approach when the harmonic voltage distortion is present in the system. The power factor is generally worse than the precalculated value based on fundamental frequency analyses[3]. In addition, depending on the short circuit capacity at the load bus, the calculated capacitor value could result in a harmonic resonant condition. (2) The second approach takes the harmonic distortion into account but assumes that the transmission line impedance (source impedance) is negligible in the harmonic analyses . In the practical case, voltage drops do occur and the value of 'optimal' capacitor and the resulting power factor is no longer optimal as expected.

Other points worth mentioning are that:(1) harmonics caused by source distortion are always arrested by detuning the harmonic resonance circuit by modifying the compensating capacitor value. (2) harmonics generated locally by nonlinear load are arrested by tune filters installed to prevent the currents from being injected into the system [1]. However. such filters are resorted to only for heavily nonlinear loads because of their associated high cost.

As a result of the draw-backs of these conventional approaches mentioned above. the present work seeks to consider harmonics generated from both the equivalent source and the load. The load harmonics are assumed not to be sufficiently serious to suggest tune filters but when combined with source harmonics, the use of only a pure capacitive compensator would degrade power factor and overload equipment.

The objective of this thesis is to develop solution procedures as well as mathematical formulations for calculating the optimal shunt compensators - both capacitor (C) and capacitor-inductor (LC) compensators - for the cases of linear loads for nonsinusoidal source with non zero source impedance and nonlinear loads for nonsinusoidal source with non zero source impedance. The goal of this work is to meet three basic criteria : minimising transmission line losses, maxmising the power factor (also known as the external efficiency), and maximising the transmission efficiency (also known as the internal efficiency) under the just mentioned operation conditions. The capacitor value chosen is constrained by the values that produce resonance. The term *resonance* is defined as an operating condition such that the magnitude of the impedance of the circuit passes through an extrmum (i.e., maximum or minimum). For example, a simple series LC circuit has impedance

$$Z(\omega)=j\omega L+\frac{1}{j\omega C} \tag{1.1}$$

The magnitude of this function has a minimum which is $|z|=0$ *at*

$$\omega = \frac{1}{\sqrt{LC}} \tag{1.2}$$

An extremum (maximum) occurs at $\omega \to \infty$. When resonance occur at a particular harmonic frequency, the capacitors and the associated circuit may be subjected to current and voltage overloading. In extreme cases. the current overloading can be up to ten times greater than the nominal line current.

This work is divided into chapters of which the present discussion forms the chapter one - an introduction to the work. The rest of the chapters deal with a specific general problem leading to the main problem of optimal shunt compensators at nonlinear busbars. Chapter two is devoted to the contemporary definition of power terms in sinusoidal and nonsinusoidal conditions with particular emphases on the definition of the reactive power since its measurement and control requires that it is suitably defined. Chapter three is on capacitive shunt compensation of linear loads with sinusoidal source with respect to the three mentioned criteria. Chapter four treats shunt compensators for linear nonsinusoidal circuits. Chapter five considers nonlinear load with nonsinusoidal source voltage and the optimal shunt L-C compensators required. Chapter six concludes this thesis.

CHAPTER TWO

LITERATURE REVIEW AND DEFINITION OF POWER TERMS.

2.0 Introduction

As a result of the development of efficient. high-power. semi-conductor switching devices which are widely used for the control of large electrical machines and industrial processes. electric utilities are faced with problems associated with the measurement of energy flow and optimum use of transmission networks. Under these operational conditions. the need to know accurately both the active power being delivered to the load under highly nonsinusoidal conditions and the means to determine and control the reactive current so that losses in the network can be minimised. are vital to the electrical utilities. To measure and control the reactive power requires that it should be suitably defined . This chapter is therefore devoted for the literature review of power factor compensations and the definitions of power terms in both sinusoidal and nonsinusoidal conditions. The review of the definitions of terms under sinusoidal conditions are the generally accepted definitions, however , for the nonsinusoidal case the definition of terms have no general consensus[4]. Various reseach engineers have given different definitions for the reactive power under nonsinusoidal condition .

2.1 Literature Review

The current status of reactive power compensation for power factor improvement are as follows:

The first is the use of shunt capacitors with the assumption of sinusoidal source with linear loads which have found extensive usage in all utilities since time in memorial [3-4] . In this approach, loadflow analysis is carried out at each busbar. The voltage levels are checked for practical limits and reactive Kilovoltamperes (KVAR) are installed at busbars with poor power factor. With this assumption it is easy to find the value of the shunt capacitor corresponding to the required KVAR. The second approach which emerged during the early 1980's is the use of shunt capacitors for power factor correction of slightly distorted busbars voltages [4-14]. References 4 to 14 have all dealt extensively with this topic of shunt capacitor compensation in the presence of busbars voltage distortions with the outcome that the neglection of these distortions result in inaccurate representation of the compensation. However, it should be made clear that this second approach also assums a linear load . The mathematical modelling leads to nonlinear equations in one variable - the capacitor (C). Nonlinear optimisation techniques are used to solve these equations in order to obtain the optimal shunt capacitor for compensations. References 4 and 5 in particular obtained three nonlinear equations in C for the line losses, power factor, and the transmission line effeciency. Each of these equations is optimised independently as an objective function using the Golden section search algorithm . This has been paralleled with recent definitions of the reactive power and its physical interpretations as contained in references 15 to 21.

Recent literature considers the effect of nonlinear loads on the value and location of the optimal capacitor as found in references 22 to 25. The procedures used by these references accounted for harmonics injected by nonlinear loads in the capacitor placement problem. Based on some simplifying assumptions, capacitor selection and system modeling at both fundamental and harmonic frequencies are derived. In all of these references, the objective function to be maximised consist of the net dollar savings due to energy and peak power loss reduction after subtracting the shunt capacitor cost. The objective function is a one variable problem in C (the capacitor).

The final stage is the consideration of an optimal L-C compensator for nonlinear loads under nonsinusoidal situation which so far has been documented only by reference 1. In this reference, the shunt compensator is in the form of a series inductor (L) and capacitor (C) . The objective function is a two variable problem in L and C. A model of the system is derived from the basic circuit theory of an equivalent single phase network. The derivation takes into consideration the source impedance, source voltage distortion and the effect of the nonlinear load at harmonic frequencies. By this approach, theoretical expressions are obtained for both the source current and the power factor as functions of the two variables L and C. A microcomputer based parameter estimator is used to experimentally determine the distribution characteristics of these random variables at a supply bus.

In a nutshell the shunt capacitor compensators have been treated under all conditions and a new trend of the approach which is the optimal L-C compensators is introduced where both the equivalent source and load are considered to generate harmonics. It is found out that only reference 1 addressed the issue of nonlinear load in terms of L - C compensators and it did so only for cost analysis . The optimisation results are such that for any given utility discount factor, the optimal LC compensator corresponding to maximum average discount and minimum compensator cost is determined.

Nevertheless these researches are no means exhaustive treatment of the subject of reactive power compensation for power factor improvement. Therefore the main theme of this thesis is to consider power factor correction at busbars of linear and nonlinear loads in the presence of nonsinusoidal source voltage.

2.2 Definition of Terms in Sinusoidal Circuits

Power:

Time rate of energy transfer or energy utilisation.

Instantaneous Power (p) :

Power delivered at any instant of time. Its dimension is the voltamperes and has the physical nature of power. It satisfies the principle of conservation of energy [3].

Average or Active Power (P_s):

It is the average value of the instantaneous power. Its dimension is the voltamperes with a physical nature of power and satisfies the principle of conservation of energy. It is the value that would be measured by a wattmeter connected in the circuit.

Apparent Power (S_s):

It is the product of the rms voltage and rms current and has the dimension of voltamperes . It represent the maximum energy transfer capability of a system, It is also referred to as " apparent voltamperes ". It does not satisfy any conservation principle.

Reactive Power (Q) :

It is defined as " that component of apparent voltamperes that is created by a combination of 90° out-of-phase current components with corresponding voltage components " [3].Its dimension is the voltamperes. It has been considered for long as not having any physical nature [3] but recent research papers point to the opposite [15-16] as will be discussed. It is not associated with energy dissipation and may or may not be associated with the storage of energy in electric or magnetic fields of force. It does not satisfy any conservation principle and in linear sinusoidal circuits, it is entirely related to energy storage and its components can be summed algebraically.

Root Mean Square (rms):

The root mean square (rms) or effective value of a mathematical continuous and periodic function $f(\omega t)$ of period T is defined by the relation :

$$F_s = \left\{ \frac{1}{T} \int_0^T f^2(\omega t) d\omega t \right\}^{\frac{1}{2}} \qquad (2.1)$$

If $f(\omega t)$ is defined as:

$$f(\omega t) = F_m \sin(\omega t \pm \psi) \qquad (2.2)$$

Where F_m is the maximum value of $f(\omega t)$, then the rms value F_s is given as:

$$F_s = \frac{F_m}{\sqrt{2}} \qquad (2.3)$$

For example the rms current and voltage are given respectively as

$$I_s = \left\{ \frac{1}{2\pi} \int_0^{2\pi} i_s^2(\omega t) d\omega t \right\}^{\frac{1}{2}} \qquad (2.4a)$$

and

$$E = \left\{ \frac{1}{2\pi} \int_0^{2\pi} e^2(\omega t) d\omega t \right\}^{\frac{1}{2}} \qquad (2.4b)$$

Power factor (PF)

It is the ratio of the average power entering the circuit to the product of rms voltage and rms current at the circuit terminals (ie the apparent power).

$$PF = \frac{average \;\; power}{apparent \;\; voltamperes}$$

$$= \frac{P_s}{S_s} = \frac{P_s}{EI_s} \qquad (2.5)$$

Distortion Power (D_c):

Arises only in a nonsinusoidal system as a result of the combination of voltages and currents of different harmonics. It is found in the expression

$S_s = P + Q + D_c$ (2.6)

Where the arrows denote vector (or phasor) quantities.

Nonreactive Power (N) :

Defined as

$$N^2 = S^2 - Q^2$$

$$= P^2 + D^2_c \qquad (2.7)$$

2.3 Definition of Power Terms in Nonsinusoidal Circuits

Instantaneous Voltage (e)

It is the voltage at any instant of time. described by the equation

$$e = \sum_{k=0}^{\infty} E_{km}\sin(k\theta + \psi_k) \qquad (2.8)$$

Where k is voltage harmonic number , k = 0 , 1, 2 ,......

and $\theta = \omega t = 2\pi F t$

and m denotes the maximum

Instantaneous Current (i)

It is the current at any instant of time , described by

$$i = \sum_{n=0}^{\infty} I_{nm}\sin(n\theta + \Delta_n) \qquad (2.9)$$

where n = 0, 1, 2 , ... and n is not equal to k.

Instantaneous Power (p)

This is the power at any time , it is expressed as;

$$p = ei \qquad (2.10)$$

RMS or effective voltage and current

$$E = \left\{ \frac{1}{2} \sum_{k=0}^{\infty} E_{km}^{2} \right\}^{\frac{1}{2}}$$

$$= \left\{ \sum_{k=0}^{\infty} E_{k}^{2} \right\}^{\frac{1}{2}} \tag{2.11}$$

Where E_{km} is the maximum value of the K^{th} harmonic voltage and E_k is the rms value of E_{km} .

Similarly

$$I = \left\{ \frac{1}{2} \sum_{n=0}^{\infty} I^{2}_{nm} \right\}^{\frac{1}{2}}$$

$$= \left\{ \sum_{n=0}^{\infty} I^{2}_{n} \right\}^{\frac{1}{2}} \tag{2.12}$$

Active Power (P)

$$P = \frac{1}{T}\int_{0}^{T} p\,dt$$

$$P = \sum_{k=0}^{\infty} E_k I_k \cos\Phi_k \tag{2.13}$$

Reactive Power (Q)

The conventional definition of power terms under nonsinusoidal condition was first proposed by Budeanu in 1927. He was the first to think of this nonsinusoidal power in terms of active power P , reactive power Q , distortion power D , and apparent power S [17] . Since then many innovative definitions of power components have been continuously proposed . A few of these proposed definitions have been tabulated in table 2.1 with the expressions labled and the names of researchers who proposed them attached. Their approaches can be presented briefly as follows :

(1) Fryze's definition :-

This was proposed in 1932 and the approach was to divide the distorted current into two components - the active current $i_a(t)$ and the reactive current $i_r(t)$. Equations 8 and 9 on table 2.1 define these currents and the related power is defined by equations 10 and 11 on the table [17].

(2) Shepherd and Zakihani's definition : -

This definition was proposed in 1972 when these authors believed that the definition of reactive power given by equation 5 on table 2.1 was based on a fallacy . Their defintions are given on the table by equations 12 - 15 .

(3) Sharon's definition :-

In 1973 Sharon considered the possibility of discontinuity of equation 4 in the reactive power compensation by linear devices. He then proposed equations 16 and 17 of table 2.1 as modification of equation 4 .

(4) Emanuel's definition :-

In his definition, Emanuel , from a physical viewpoint considered apparent power to have only two components - the active power given by equation 4 and the complementary power given by equation 18 both found in table 2.1.

(5)Kuster and Moore's definition :-

An innovative definition for power - component calculation was formulated by dividing the current into active i_p , inductive i_{ql} , and residual i_{qlr} (or capacitive reactive current i_{qc} and residual capacitive current i_{qcr}) [18-19]. The power components corresponding to each current are given by equations 24 - 27 in table 2.1 .

(6) Czarneki's definition :-

Czarneki's equations for calculating the power components under nonsinusoidal conditions are given by equations 34 - 38 of table 2.1 [8-11]. The V_m of equation 1 was neglected, and Y_n was the equivalent admittance of the n th - order harmonic. G_e was the equivalent conductance of the load . i_a was the active current. i_r was the reactive current. i_s was the scattered current . and i_g was the generated current .

Table 2.1 : Survey of power-component definitions

Proposer (year)	Voltage	Current		
			Apparent power	Active p
Budeanu (1927)	$v(t)=\sum\sqrt{2}V_n\sin(n\omega t+\alpha_n)$ $+\sum\sqrt{2}V_m\sin(\omega_m t+\beta_m)$ (1)	$i(t)=\sum\sqrt{2}I_n\sin(n\omega t$ $+\alpha_n+\varphi_n)$ $+\sum\sqrt{2}I_g\sin(\omega_g t+\beta_g)$ (2)	$S=V_{rms}I_{rms}$ $=(\sum V_n^2+\sum V_m^2)^{\frac{1}{2}}$ $x(\sum I_n^2+\sum I_g^2)^{\frac{1}{2}}$ (3)	$P=\frac{1}{T}\int_0^T$ $=\sum V$ (4)
Fryze (1972)	same as Eq.(1)	$G=\frac{P}{V_{rms}^2}$ (7) Active current $i_a(t)=Gv(t)$ (8) Reactive current $i_r(t)=i(t)-i_a(t)$ (9)	same as Eq.(3)	$P=V_{rms}$ (10) same as
Shephard and Zakihani (1972)	same as Eq. (1)	same as Eq. (2)	$S^2=S_R^2+S_X^2+S_D^2$ (12)	(15) Same as
Sharon (1973)	same as Eq. (1)	same as Eq. (2)	same as Eq. (3)	

		Power component	
pparent power	Active power	Reactive power	Distortion power
$=V_{rms}I_{rms}$ $=(\sum V_n^2+\sum V_m^2)^{\frac{1}{2}}$ $(\sum I_n^2+\sum I_g^2)^{\frac{1}{2}}$	$P=\frac{1}{T}\int_0^T v(t)i(t)dt$ $=\sum V_n I_n \cos\varphi_n$	$Q=\sum V_n I_n \sin\varphi_n$	$D=(S^2-P^2-Q^2)^{\frac{1}{2}}$
)	(4)	(5)	(6)
me as Eq.(3)	$P=V_{rms}I_a$ (10)	$Q=V_{rms}I_r$ (11)	N.A.
$=S_R^2+S_X^2+S_D^2$ (12)	same as EQ. (4) (15)	**Active apparent power** $S_R^2=\sum V_n^2\sum(I_n\cos\varphi_n)^2$ (13) **True reactive power** $S_x^2=\sum V_n^2\sum(I_n\sin\varphi_n)^2$ (14) **Distortion power** $S_D^2=\sum V_n^2\sum I_g^2+\sum V_m^2(\sum I_n^2+\sum I_g^2)$	
me as Eq. (3)	Same as Eq. (4)	**Quadrature reactive power** $S_Q^2=V_{rms}^2\sum I_n^2\sin^2\varphi_n$ (16) **Complementary power**	

Emanual (1974)	**same as Eq. (1)**	**same as Eq. (2)**	**same as Eq. (3)**	**Same as**
Kuster & Moore (1980)	$v(t)=\frac{dv(t)}{dt}$ **(19) -** $v(t)=\int v(t)dt$ -	$i_a(t)=\frac{v(t)}{V_{rms}^2}\{\frac{1}{T}\int_0^T v(t)i(t)dt\}$ - **(21)** **Inductive reactive current** $i_{q}=\frac{v(t)}{V_{rms}^2}\{\frac{1}{T}\int_0^T v(t)i(t)dt\}$ - **(22a)** **Residual inductive reactive current** $i_{qr}=i-i_p-i_q$ - **(23a)** **or** **Capacitive reactive current** $i_{qc}=\frac{v(t)}{V_{rms}^2}\{\frac{1}{T}\int_0^T v(t)i(t)dt\}$ - **(22b)** **Residual capacitive reactive current** $i_{qcr}=i-i_p-i_{qc}$ - **(23b)**	$S^2=P^2+Q_c^2+Q_{cr}^2$ **(24)**	$P=V_{rms}$ $P=V_{rms}$ **Same as**
Czarneki (1983)	**same as Eq. (1)** except for V_m -	$Y_n=\frac{I_n}{V_n}=G_n+jB_n$ - (28)	$S^2=P^2+Q^2+Q_s^2+Q_g^2$ (34)	

e as Eq. (3)	Same as Eq. (4)	$S_c^2=S^2-P^2-S_Q^2$ (17) **Complementary power** $P_c^2=S^2-P^2$ (18)	
$=P^2+Q_c^2+Q_{cr}^2$)	$P=V_{rms}I_a$ (25)	**Inductive reactive power** $Q_i=V_{rms}I_{qi}$ (26a) **or** **Capacitive reactive power** $Q_c=V_{rms}I_{qc}$ (26b)	**Residual inductive reactive power** $Q_{ir}=V_{rms}I_{qir}$ (27a) **or** **Residual capacitive reactive power** $Q_{cr}=V_{rms}I_{qcr}$ (27b)
	$P=V_{rms}I_a$ (35)	$Q=V_{rms}I_r$ (36)	**Scattered power** $Q_s=V_{rms}I_s$ (37) **Generated power** $Q_g=V_{rms}I_g$ (38)
	Same as Eq. (4)	**Harmonic apparent power** $S_h^2=\sum V_n^2I_n^2$ (40)	**Total distortion power** $D_T^2=\sum V_n^2I_g^2$ (41)
$=P^2+Q^2+Q_s^2+Q_g^2$ 4)			

(7) Slonim and Wyk's definition :-

Equation 39 - 41 were the equations proposed by Slonim and Wyk [15] Detailed discussion of their approach is given below.

$$Q = \sum_{k=1}^{\infty} E_k I_k \sin\Phi_k \tag{2.14}$$

In equations (2.13) and (2.14) it is assumed that the voltage and the current each consist of k harmonic components , that is , k = n .

Apparent power (S)

$$S = EI \tag{2.15a}$$

$$S^2 = P^2 + Q^2 + D_r^2 \tag{2.15b}$$

Where D_r arises as a result of voltages and currents of different harmonics. that is $k \neq n$.

From (2.15a) , (2.11), and (2.12)

$$S^2 = E^2 I^2$$

$$= \sum_{K=0}^{\infty} E^2{}_k \sum_{n=0}^{\infty} I^2_n \tag{2.16}$$

Equation (2.16) can also be written in the form ,

$$S^2 = \sum_{k=0}^{\infty} E^2{}_k I^2{}_k + \sum_{k=0,\, n=0,\, k \neq n}^{\infty} E^2{}_k I^2{}_n \tag{2.17}$$

$$= S^2{}_h + D^2{}_{\Sigma} \tag{2.18}$$

The first part of equation (2.17) has the same form and the same physical nature as the apparent power of the particular harmonic of equation (A.15), hence the first part can be written as :

$$S^2{}_h = \sum_{k=0}^{\infty} S^2{}_k \tag{2.19}$$

Where $S_k = E_k I_k$ (2.20)

Each component of $E_k I_n$ in the second part of (2.17) depends on the effective values of the voltage and current , having different frequancies . therefore each component may be called the distortion power of the particular pair of harmonics. D_{kn} . defined as :

$$D_{kn} = E_k I_n \tag{2.21}$$

Then. the second term of (2.17) may be called the total distortion power D_r . defined as :

$$D_r = \sum_{k=0,\, n=0,\, k\neq n}^{\infty} E^2_k I^2_n = \sum_{k=0,\, n=0,\, K\neq n}^{\infty} D^2_{kn} \tag{2.22}$$

Assumpting a nonsinusoidal source with linear circuit , where

$E_k = z_k I_k$, and $z^2_k = r^2 + x^2_k$

The following terms of S_k and D_{kn} are useful :

$$S^2_k = E^2_k I^2_k = z^2_k I^4_k = r^2 I^4_k + x^2_k I^4_k$$

$$= P^2_k + Q^2_k \tag{2.23a}$$

$$D^2_{kn} = E^2_k I^2_n = z^2_k I^2_k I^2_n = r^2 I^2_k I^2_n + x^2_k I^2_k I^2_n$$

$$= P^2_{kn} + Q^2_{kn} \tag{2.23b}$$

The apparent power p = ei could be expressed in the forms ;

(a) $\sum_{k=0}^{\infty} E_{km} \sin(k\theta + \psi_k) \sum_{k=0}^{\infty} I_{nm} \sin(n\theta + \Delta_n)$

(2.24)

$\theta = \omega t$

from (A.8) and (A.9)

(b) $\sum_{k=0,\, k=n}^{\infty} E_k I_k \{\cos\Phi_k - \cos(2k\theta + \psi_k - \Delta_k)\}$

$+ \sum_{k=0,\, n=0,\, k\neq n}^{\infty} E_k I_n \{\cos((k-n)\theta + \psi_k - \Delta_n) - \cos((k+n)\theta + \psi_k + \Delta_n)\}$

(2.25a)

from the expansion of (2.24)

$$(c)\quad \sum_{k=0,\,k=n}^{\infty} E_k I_k\{\cos\Phi_k - \cos(2k\theta + \psi_k + \Delta_k)\}$$

$$+ \sum_{k=0,\,n=0\,k\neq n}^{\infty} E_k I_n\{\cos\Phi_k - \cos((k+n)\theta + \psi_k + \Delta_n)\}$$

$$- \sum_{k=0,\,n=0\,K\neq n}^{\infty} E_k I_n\{\cos\Phi_k - \cos((k-n)\theta + \psi_k - \Delta_n)\} \qquad (2.25b)$$

Form (b) consist of $P_k = E_k I_k \cos\Phi_k$ as well as $P_{kn} = E_k I_n \cos\Phi_k$ similar to form (c). This allows understanding that in the frequency domain all components of apparent power S_k and D_{kn} may be active and reactive components. Hence D^2_{kn} is similar to S^2_k and may be described by P^2_{kn} and Q^2_{kn} .

Substituting (2.23) into (2.22). (2.19). and (2.18), the new expression for apparent power in a nonsinusoidal system is obtained as :

$$S^2 = \sum_{k=0\ n=0}^{\infty} P^2_k + \sum_{k=0,\,n=0,\,K\neq n}^{\infty} P^2_{kn} + \sum_{k=0,\,k=n}^{\infty} Q^2_k + \sum_{k=0,\,n=0\,k\neq n}^{\infty} Q^2_{kn}$$

$$= P^2_{\Sigma} + Q^2_{\Sigma} \qquad (2.26)$$

Equations (2.26) is a very importantant experession. It shows that S may be represented either as a many - dimentional vector with components S_k and D_{kn} , or as a two - dimentional vector with components P_{Σ} and Q_{Σ} , in constrast with previous proposals. This form allows the use of the standard units VA , W , and VAR .

2.3.1 Distortion Power

(a) If the spectra of the voltage and the current waveforms have the same harmonics, then using (2.23b) and (2.18) will result in the following expressions:

$$S^2 = r^2 \sum_{k=0}^{\infty} I^4_k + r^2 \sum_{k=0,\, n=0,\, k \neq n}^{\infty} I^2_k I^2_n + \sum_{k=0}^{\infty} x^2_k I^4_k + \sum_{k=0,\, n=0,\, k \neq n}^{\infty} x^2_k I^2_k I^2_n$$

$$= r^2 \left\{ \sum_{k=0}^{\infty} I^2_k \right\}^2 + \left\{ \sum_{k=0}^{\infty} x_k I^2_k \right\}^2 + \sum_{k=0,\, n=0,\, k \neq n}^{\infty} (x_k - x_n)^2 I^2_k I^2_n$$

$$= P^2 + Q^2 + D^2_c \qquad (2.27)$$

Taking into consideration that $P_k = r I^2_k$, and $Q_k = x_k I^2_k$, it is conclusive that :

$$D_c = \sqrt{\sum_{k=0}^{\infty} (x_k - x_n)^2 I^2_k I^2_n} \qquad (2.28)$$

(2.28) is the straight forward expression of the distortion power D_c (in the terminology of current theory) . It has a purely reactive character which suggests that the definition of distortion power as nonreactive power is questionable .

(b) If the spectra of the voltage and the current waveforms have different harmonics , then (2.26) may be further analysed as follows:

$$E^2 = \sum_{k=0}^{\infty} E^2_h = \sum E^2_k + \sum E^2_e \qquad (2.29a)$$

$$I^2 = \sum_{n=0}^{\infty} I^2_n = \sum I^2_k + \sum I^2_l \qquad (2.29b)$$

$$S^2 = E^2 I^2 \quad = \sum E^2_k \sum I^2_k + \sum E^2_e \sum I^2_k + \sum E^2_e \sum I^2_l + \sum E^2_k \sum I^2_l \qquad (2.30)$$

The first term of (2.30) corresponds to the apparent power created by voltage and current harmonics having the same frequency. Hence, the result must be identical to (2.27). The last three parts of (2.30) create distortion power, because the components $E_e I_k$, $E_e I_l$ or $E_k I_l$ are products of voltage and current having different frequencies and they are similar to those in (2.21). Using (2.23b), the distortion power which is the sum of these three terms may be obtained as:

$$D^2_i = r^2\Big\{\sum_{e\neq k} I^2_e I^2_k + \sum_{e\neq f} I^2_e I^2_f + \sum_{k\neq f} I^2_k I^2_f\Big\} + \sum_{e\neq k} x^2_e I^2_e I^2_k + \sum_{e\neq f} x^2_e I^2_f + \sum_{k\neq f} x^2_k I^2_k I^2_f$$

$$= P^2_i + Q^2_i \qquad (2.31)$$

It is possible to call the resulting components P_i , Q_i , and D_i , the imaginary active , reactive , and distortion powers respectively, because they appear owing to the different spectra of e and i . The actual active and reactive powers of the system are created by voltages and currents having the same harmonic spectra.

Adopting the symbols of (2.26), (2.27), (2.28), and (2.31) ,

$$S^2 = P^2 + Q^2 + D^2_c + D^2_i$$

$$= P^2 + P^2_i + Q^2 + D^2_c + Q^2_i$$

$$= P^2_\Sigma + Q^2_\Sigma \qquad (2.32)$$

Where P_Σ and Q_Σ *are total* , P , D_c and Q *are actual* , P_i and Q_i are imaginary powers respectively , and

$$P^2_\Sigma = P^2 + P^2_i$$

$$Q^2_\Sigma = Q^2 + D^2_c + Q^2_i$$

It is seen from (2.32) that S is always larger than P and also the definition of nonreactive power N in (2.7) is not correct according to analysis because of the imarginary distortion power of (2.31), which consists of both active and reactive components [15] .

2.4 Complex Power and Apparent Voltamperes

2.4.1 Definition of Complex Power

Adopting the phasor representation of the supply voltage $\bar{E}$, and the supply current $\bar{I}_s$ in figure 2.1 and assuming a linear sinusoidal system we can in cartesian form representation write

$$\bar{E} = E_r + jE_q \quad (2.33a)$$

$$\bar{I}_s = I_r + jI_q \quad (2.33b)$$

The magnitudes of the phasors are equal to the rms or effective values of the variables

$$|\bar{E}| = E = \sqrt{E^2_r + E^2_q} \quad (2.34a)$$

$$|\bar{I}_s| = I = \sqrt{I^2_r + I^2_q} \quad (2.34b)$$

We can also write the conjugate of the phasors as

$$\bar{E}^* = E_r - jE_q$$

$$\bar{I}^*_s = I_r - jI_q$$

The complex power denoted by S^*_s has two alternative . arbitrary definitions in the literature :

(1) $S^*_s = \bar{E}^*\bar{I}_s$

$$= (E_r - jE_q)(I_r + jI_q)$$

$$S^*_s = (E_rI_r + E_qI_q) - j(E_qI_r - E_rI_q) \quad (2.35a)$$

$$= P - jQ$$

Where $P = E_rI_r + E_qI_q$ and $Q = E_qI_r - E_rI_q$

This definition is recommended by the International Electrotechnical Convention (EIC) [3] and supported by most references originating in the U.S.A.

(2)

$$S^*_s = \bar{E}\bar{I}^*_s$$

$$= (E_r + jE_q)(I_r - jI_q)$$

$$= (E_r I_r + E_q I_q) + j(E_q I_r - E_r I_q) \qquad (2.35b)$$

$$= P + jQ$$

This definition of complex power is recommended and used by electric power utilities in the United Kingdom .

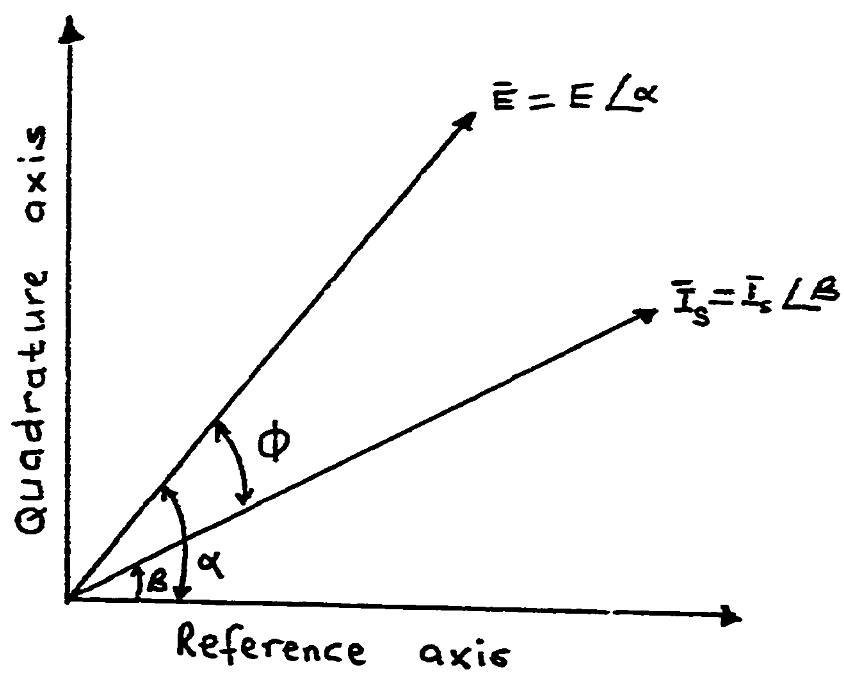


Figure 2.1 Phasor diagram

2.4.2 Complex Power and Apparent Voltamperes .

The use of the term complex power S^{*} permits algebraic summation of both P and Q at any point in the system. This summation is possible because in linear . sinusoidal systems - and only in linear . sinusoidal systems - is the reactive voltamperes associated entirely with energy storage.

$$|S^{*}_{s}| = \sqrt{P^{2}_{s} + Q^{2}_{s}} \qquad (2.36)$$

The concept of complex power S^{*}_{s} is purely mathematical and has a dimension of voltamperes but no physical existence and cannot therefore be measured [3].

It is important to note that the concept of complex power is meaningless in nonsinusoidal systems. If the supply voltage is nonsinusoidal. or the load is nonlinear. or both . then phasor representation of voltage and current is invalid and resolution of the total voltamperes into cartesian components is mathematically invalid as well as being physically meaningless.

As already defined, the apparent power S_{s} = (rms supply voltage) X (rms supply current), is a figure of merit that defines the maximum energy transfer capability of the system. Its definition is independent of waveform and it can therefore be used in discussions concerning power factor and power factor correction in nonsinusoidal systems as well as in linear sinusoidal systems.

The apparent voltamperes S_{s} and the complex power S^{*}_{s} are equal in magnitude irrespective of sign conventions.

$$|S_{s}| = |EI_{s}| = |S^{*}_{s}| \qquad (2.37)$$

While the apparent power S_{s} *is a scalar quantity*, the complex power S^{*}_{s} can be interprated . in sinusoidal circuits and systems only . as having cartesian co-ordinates equal to the average power P_s and *the reactive voltamperes* Q_s.

The definitions given for power terms in this chapter especially that of the reactive power given by Budeannu and Kuster and Moore are being used in the next subsequent chapters for the compensation discussions. In particular the next chapter employs these definitions to address the problem of compensating a linear sinusoidal circuit .

CHAPTER THREE

COMPENSATION FOR LINEAR SINUSOIDAL CIRCUITS

3.0 Introduction

This chapter is a review of the well established power factor correction of linear loads with sinusoidal supply voltage. The purpose of this review is to gradually lead the reader to the ultimate goal of this research - power factor correction of nonlinear loads with nonsinusoidal supply voltage.

Most AC electric machines draw from the supply apparent power in terms of kilovolt-amperes (KVA) which is in excess of the useful power measured in kilowatts (KW) required by the machine. The power factor of the machine is the ratio of these two quantities:

$$\frac{\text{Useful Power}}{\text{Apparent Power}} \quad \text{or} \quad \frac{KW}{KVA} = \text{Power Factor} \quad (\cos\Phi) \tag{3.1}$$

The power factor (PF) is dependent upon the type of machine in use .

Installations that are likely to result into low power factor which need correction are the same as those devices mentioned in chapter two as sources of harmonics in power systems. However , for the sake of completeness of this present discussion , the type of load with low power factor problems are repeated briefly here.

(a) Induction motors of all types (which form by far the greatest industrial load on A.C. mains).

(b) Power thyristor installations in both rectification and inversion operatin modes (for D.C. motor control and electro-chemical processes).

(c) Power transformers and voltage regulators.

(d) Welding machines

(e) Electric-arc and induction furnaces

(f) Choke coils and magnetic systems

(g) Neon signs and fluorescent lighting .

In a linear sinusoidal circuit. the method used for the improvement of power factor is the introduction of reactive kilovolt-amperes (KVAR) into the system in phase opposition to the wattless or reactive current as defined in chapter two . By doing so , this introduced KVAR effectively cancels the effect of the reactive current in the system . This is achieved usually . either by rotary machines (synchronous condensers) or static capacitors . The latter method is the most common since by virtue of its static nature. it has a longer working life and its cost may be less . This discussion will therefore focus on the use of static capacitors and since they are usally connected across the load. the term shunt capacitors is used instead of static capacitors in this work .

3.1 General Idea of Power Factor Correction

The apparent power (KVA) in an A.C. circuit can be resolved into two components vis-a-vis the in-phase component which supplies the useful power (KW). and the wattless component (KVAR) which does no useful work . Figures 3.1 and 3.2 illustrate the general idea of introducing KVAR for correcting the power factor.

$$\textit{Power Factor}\ \ PF = \cos\Phi \ = \frac{KW}{KVA} \tag{3.2}$$

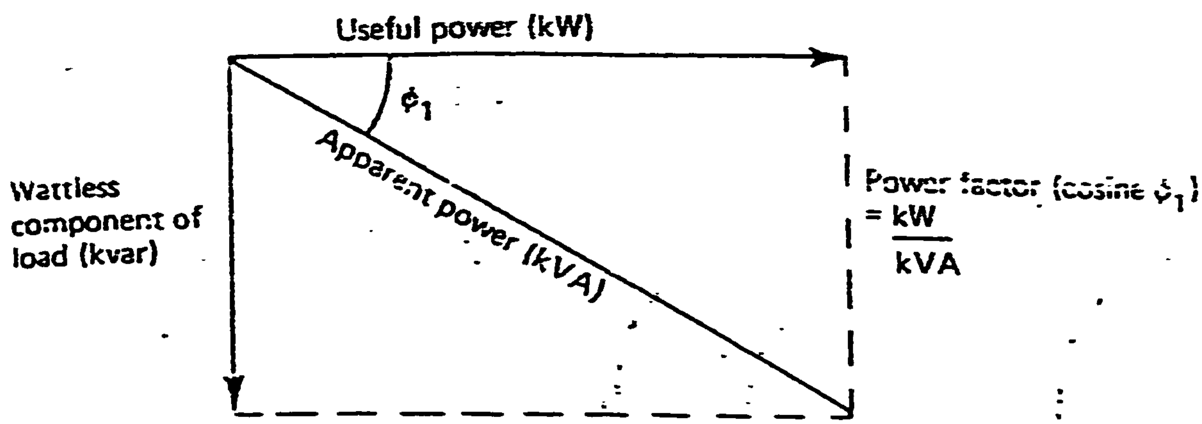


Figure 3.1 Phasor diagram of plant operating at a lagging power factor

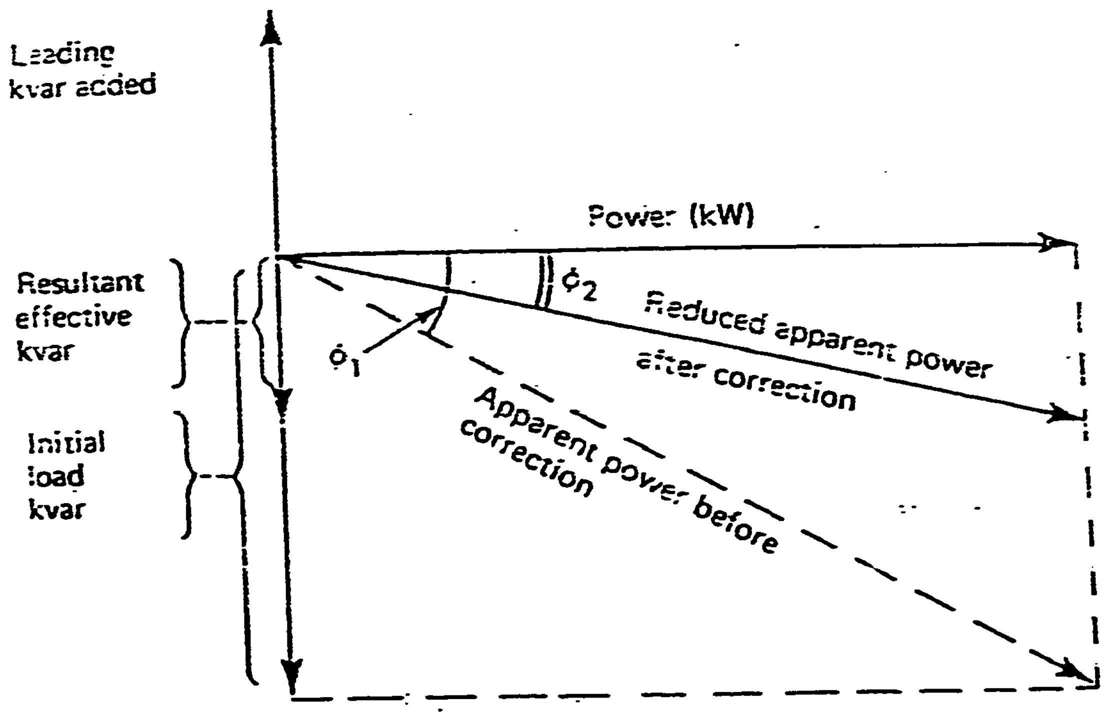


Figure 3.2 Power-factor correction by adding leading kvar

To improve the power factor . equipment drawing KVAR of approximately the same magnitude as the load KVAR. but in phase opposition (leading) is connected in parallel with the load . The points to be considered in installing either rotary phase advancers (synchronous condensers or synchronous motors) or shunt (static) capacitors for power factor correction are :

(1) Reliability of the equipment to be installed

(2) Probable life

(3) Capital cost

(4) Maintenance cost

(5) Running cost

(6) Space required and ease of installation.

3.2 Shunt Capacitor Compensation

Through out this work , analyses are carried out to find the optimum shunt compensator values that will meet three basic criteria [14] :

(i) Maximising the power factor

$$PF = \frac{P_L}{I_s V_L} \tag{3.3}$$

(ii) Minimising the line losses or line current (I^2_s) (3.4)

(iii) Maximising the transmission efficiency (η)

$$\eta = \frac{P_L}{P_s} \tag{3.5}$$

where P_L is the load active power and P_s is the source active power

3.2.1 Zero source impedance

As a beginning step , an ideal sinusoidal voltage source v_s is considered to supply current to a series R-L linear load of phase angle Φ and a pure capacitor C is connected across the load for compensation purposes as shown in figures 3.3 and 3.4 .

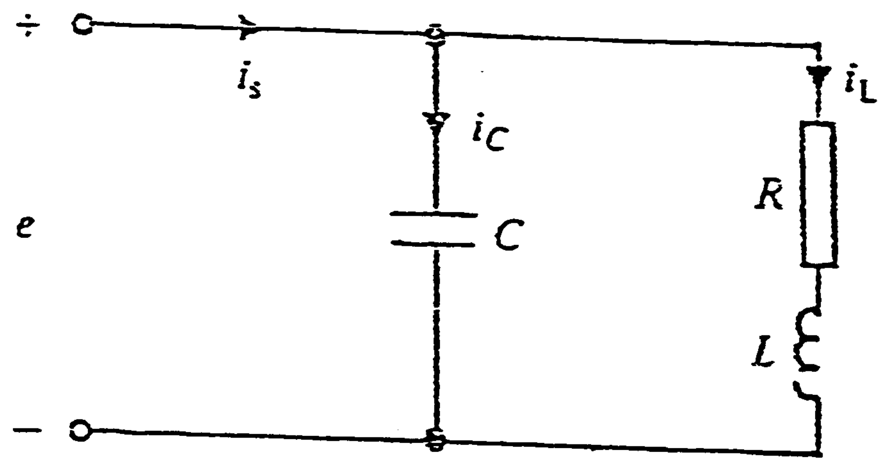


Figure 3.3 Shunt capacitor compensation of an R-L linear load

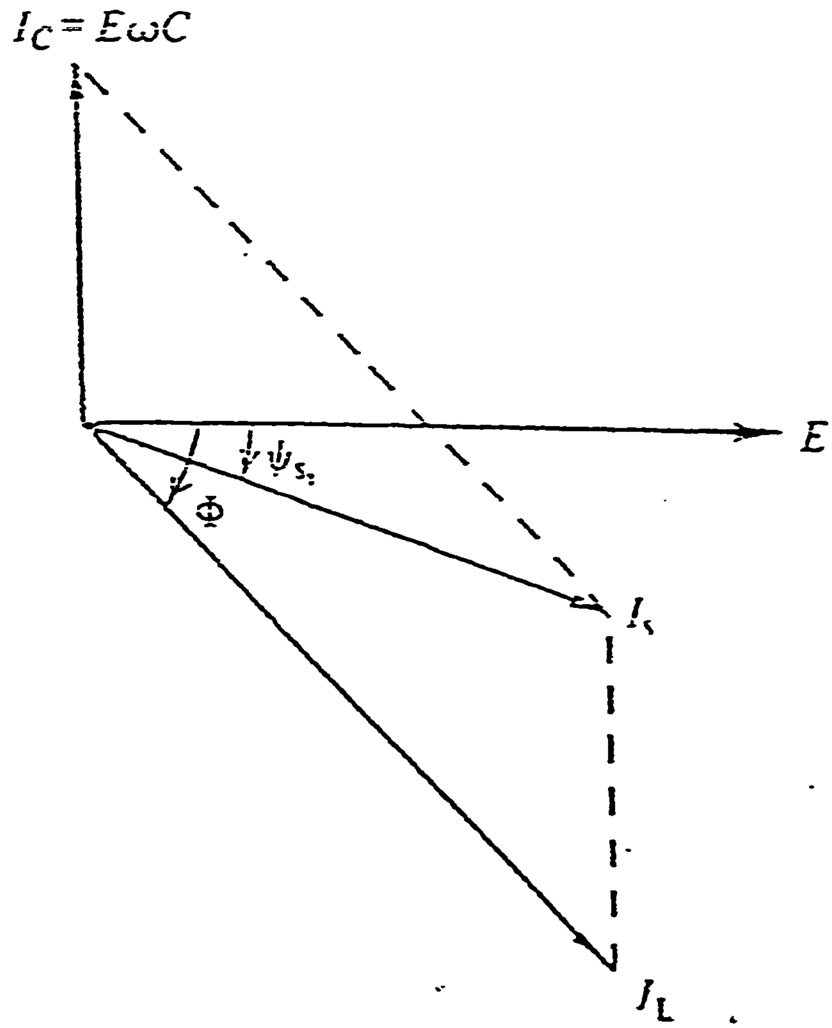


Figure 3.4 Phasor diagram of fig . 3.3

In figure 3.3 the source impedance Z_s is taken as zero hence V_s *is equal* V_L (the rms values) These voltages are expressed as :

Instantaneous source voltage

$$v_s = V_m \sin\omega t = \sqrt{2} V_s \sin\omega t \qquad (3.6)$$

Where V_m is the maximum value of v_s

Instantaneous load current (i_L)

$$i_L = \frac{V_m}{|Z|}\sin(\omega t - \Phi) = \sqrt{2} I_L \sin(\omega t - \Phi) \qquad (3.7)$$

Where $|Z| = \sqrt{R_L + X_L}$ (3.8a)

is the load impedance and I_L is the rms load current . Also

$$X_L = \omega_o L = 2\pi f L \qquad (3.8b)$$

f is the supply frequency . L is the inductance of the load and R_L *is the resistance of the load*.

Capacitor instantaneous current (i_C)

$$i_C = \frac{V_m}{|X_C|}\sin(\omega t + 90°) = \sqrt{2}\omega_o V_L C \sin(\omega t + 90°) \qquad (3.9)$$

$$\omega_o = 2\pi f$$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{\omega_o C}$ the capacitive reactance .

By Kirchhoff's current law (KCL),

$$i_s = i_L + i_C$$

$$= \sqrt{2}\{I_L \sin(\omega t - \Phi) + \omega_o V_L C \sin(\omega t + 90°)\}$$

$$= \sqrt{2} I_s \sin(\omega t + \psi_{s1}) \qquad (3.10)$$

$$I^2_s = (\omega_o V_L C)^2 + I^2_L - 2\omega_o V_L I_L C \sin\Phi \qquad (3.11a)$$

$$= (\omega_c V_L C - I_L \sin\Phi)^2 + I^2_L \cos\Phi \qquad (3.11b)$$

$$\tan\psi_{s1} = \frac{\omega_c V_L C - I_L \sin\Phi}{I_L \cos\Phi} \qquad (3.12)$$

Where ψ_{s1} is the angle between the supply current and voltage.

The power factor seen from the supply terminals is

$$PF = \frac{P_s}{S_s} = \frac{P_s}{V_s I_s} \qquad (3.13)$$

The average power P_s entering the circuit of figure 3.3 is not affected by the presence of the shunt capacitance and retains thesame value at the load .

$$P_L = P_s = V_L I_L \cos\Phi = V_s I_s \cos\psi_{s1} \qquad (3.14)$$

The lagging reactive voltamperes of the load are complementary to the average load power .

$$Q_L = V_L I_L \sin\Phi \qquad (3.15)$$

Figure 3.5 shows the analytical components of the apparent voltamperes for a capacitor compensated series R-L load.

$$S^2_s = P^2_s + Q^2_s$$

$$S^2_L = P^2_L + Q^2_L$$

$$S^2_C = P^2_C + Q^2_C \qquad (3.16)$$

Since the capacitor can be pressumed lossless $P_C = 0$ and $S_C = Q_C$.

Table 3.1 Contains components of the apparent voltamperes for capacitor compensation of a linear R-L load .

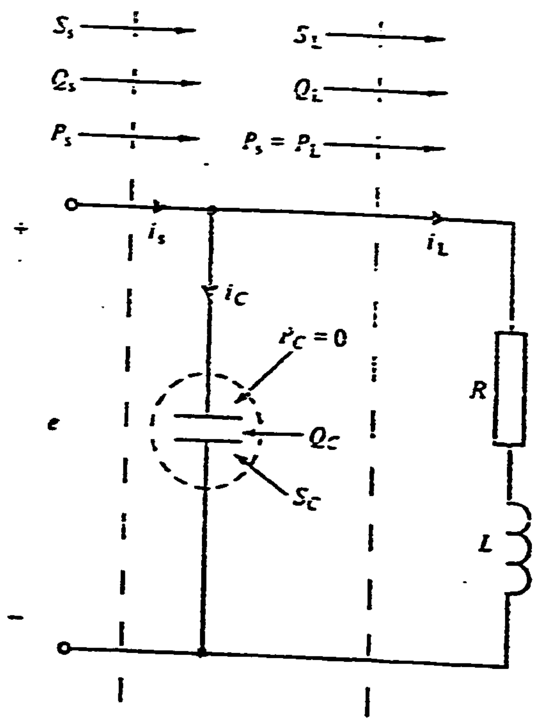


Figure 3.5 Analytical components of the apparent voltamperes for a capacitor compensated series R-L load.

TABLE 3.1 : Components of the apparent voltamperes for capacitance compensation of a linear R-L load .

Quantity	Supply	Capacitor	Load
Average Power (P)	$V_L I_L \cos\varphi$ - or $V_s I_s \cos\psi_{sl}$ -	0 -	$V_L I_L \cos\varphi$
Reactive Power (Q)	$V_s I_s \sin\psi_{sl}$ -	$\omega_o V_L^2 C$ -	$V_L I_L \sin\varphi$
Apparent Power (S)	$V_s I_s$ -	$\omega_o V_L^2 C$ -	$V_L I_L$

Combining equations (3.11) . (3.13) and (3.14) we obtain

$$PF = \frac{P_s}{VI_s} = \frac{V_L I_L \cos\Phi}{V_L\sqrt{(\omega_o V_L C - I_L \sin\Phi)^2 + I^2{}_L \cos^2\Phi}}$$

$$PF = \frac{I_L \cos\Phi}{\sqrt{(\omega_o V_L C - I_L \sin\Phi)^2 + I_L \cos^2\Phi}} \tag{3.17}$$

The terminal power factor has its maximum value at unity when the brackted term in (3.17) is zero . That is

$$\omega_o V_L C - I_L \sin\Phi = 0 \tag{3.18}$$

The optimum capacitor value C_o of C that results in maximum power factor operation can be deduced directly from (3.17) or (3.18) . It can also be obtained by differentiating (3.16) with respect to C and equating the result to zero . In both cases we get

$$C_o = \frac{I_L \sin\Phi}{\omega_o V_L} = \frac{\sin\Phi}{\omega_o |Z|} \tag{3.19}$$

$$\sin\Phi = \frac{X_L}{Z} \tag{3.20}$$

substituting equation (3.20) into (3.19) we obtain

$$C_o = \frac{X_L}{\omega_o Z^2} = \frac{X_L}{\omega_o (R^2{}_L + X^2{}_L)}$$

But $B_L = \dfrac{X_L}{R^2{}_L + X^2{}_L}$ and $\omega_o = 2\pi f$

Hence

$$C_o = \frac{B_L}{\omega_o} = \frac{B_L}{2\pi f} \tag{3.21}$$

Equation (3.21) is the value that realises unity power factor when source impedance and harmonics are neglected.

For minimising the line current we differentiate (3.10) which is rewritten here as :

$I_s = I_L + I_c$

$$= (G_L - jB_L)V_L + j\omega_o C V_L \tag{3.22}$$

where $G_L = \dfrac{R_L}{R^2_L + X^2_L}$

$$I^2_s = G^2_L + (\omega_o C - B_L)^2 V^2_L \tag{3.23}$$

$$\frac{dI^2_s}{dC} = 2\omega_o(\omega_o C - B_L)V^2_L = 0$$

$$\omega_o C_o - B_L = 0$$

$$C_o = \frac{B_L}{\omega_o} \quad or$$

$$C_o = \frac{B_L}{2\pi f} \quad \text{as in (3.21)}$$

The third condition is the maximum efficiency and since we are considering zero source impedance, maximising the efficiency is the same as minimising the line losses I^2_s (or the line current) .

Hence, for a sinusoidal source with zero source impedance, the same value of optimal capacitor C_o realises both unity power factor and minimum line current .

$$C_o = \frac{B_L(1)}{2\pi f}$$

Where $B_L(1)$ is the fundamental susceptance of the system which is represented for short as B_L .

From the viewpoint of energy transfer. an uncompensated lagging load involves the oscillation of energy between the supply and the magnetic field of the load .This energy oscillation causes an increase in the rms supply current I_s . compared with resistive load operation and thereby results in power loss in the transmission system. The action of the shunt connected capacitor is to supply the necessary lagging voltamperes required by the load as mentioned earlier on. When the power factor is corrected . the energy oscillation takes place between the capacitor and the load rather than between the supply and the load [3] . The shunt capacitor is usually connected to a busbar or to the tertiary winding of a main transformer.

3.2.2 Non Zero Source Impedance

If the source impedance is taken into consideration. the symbols on figure 3.6 have the following meanings;

R_T , X_T = Transmission system source resistance and reactance for fundamental component obtained from short circuit capacity at busbar.

R_L , X_L = Load resistance and reactance for fundamental component

C_o = optimal value of capacitor for compensation

Maximising the power factor

An optimum unity power factor occurs if the susceptance of the load B_L combined with the suaceptance of the parallel capacitor vanishes. This is the same as the case of zero source impedance .

$$B_L = \omega_o C_o \tag{3.25}$$

or power factor $PF = \dfrac{P_L}{S} = \dfrac{G_L V^2{}_L}{I_s V_L} = 1$

but $G_L V_L = I_s = V_L\{G_L + j(\omega_o C - B_L)\}$

with (3.25) being valid .

$$C_o = \frac{B_L}{2\pi f} \tag{3.26}$$

This is the same as (3.21) - meaning that whether the source impedance is taken into consideration or not, the same value of an optimal capacitor is found for maximising the power factor if and only if the ource source voltage is sinusoidal .

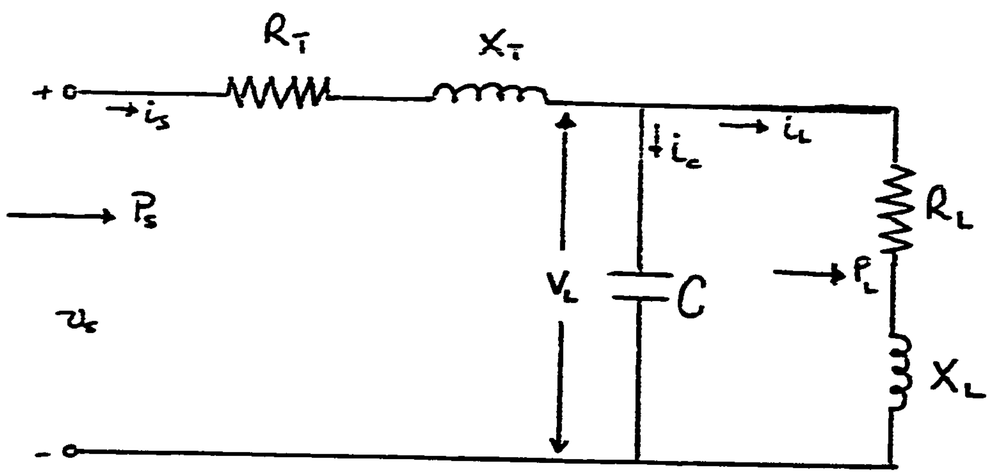


Figure 3.6 Non zero source impedance with series R-L linear load

Maximising the transmission efficiency (η)

$$\eta = \frac{P_L}{P_s}$$

where $P_L = G_L V^2_L$ and $P_s = R_T I_s^2 + P_L$

$$\eta = \frac{G_L V^2_L}{R_T I^2_s + G_L V_L^2}$$

$$= \frac{1}{1 + \dfrac{R_T}{G_L}\dfrac{I^2_s}{V^2_L}} \qquad (3.27)$$

η is a maximum if $\dfrac{V^2_L}{I^2_s}$ is maximised . Also as $Y_c = B_L$ we realise that

$$Y = Y_L + Y_c = G_L - jB_L + jY_c$$

$$= G_L \quad \text{and}$$

$$I_s = YV_L$$

$$= G_L V_L$$

or $\dfrac{V_L}{I_s} = \dfrac{1}{G_L}$ or $\dfrac{V^2_L}{I_s^2} = \dfrac{1}{G^2_L}$

Hence $$\eta_{max} = \frac{1}{1 + R_T G_L} \qquad (3.28)$$

Since the condition $Y_c = B_L$ is applied here to get the maximum efficiency which was the same condition used for getting C_o under maximum power factor it implies that maximising the power factor or maximising the transmission efficiency leads to the same optimal capacitor design .

$$C_o = \frac{B}{2\pi f}$$

Minimising the transmission loss or line current (I^2_s)

Let Z be the total impedance of the circuit of figure 3.6

$Z_T = R_T + jX_T$ transmission line impedance

$Z_L = R_L + jX_L$ load impedance

and $Z_c = j\omega_o C$ reactance of compensating capacitor

Then $Z = Z_T + (Z_L \textit{ in parallel with } Z_c)$

$$= R_T + jX_T + \frac{1}{G_l - jB_L + j2\pi fC} \tag{3.29}$$

If we let $B = 2\pi fC - B_L$ (3.30)

then Z can be rewritten as

$$Z = R_T + jX_T + \frac{1}{G_L + jB}$$

$$= \{ R_T + \frac{G_L}{G^2_L + B^2} \} + j \{ X_T - \frac{B}{G^2_L + B^2} \} \tag{3.31}$$

The modulus of the impedance is

$$Z^2 = \{ R_T + \frac{G_L}{G^2_L + B^2} \}^2 + \{ X_T - \frac{B}{G^2_L + B^2} \}^2 \tag{3.32}$$

Since $I_s \propto \frac{1}{Z}$ *or* $I^2_s \propto \frac{1}{Z^2}$

maximising Z^2 *will minimize* I^2_s hence

$$\frac{dZ^2}{dB} = 2\{ R_T + \frac{G_L}{G^2_L + B^2} \} \{ \frac{-2BG_L}{(G^2_L + B^2)^2} \}$$

$$-2\left(X_T - \frac{B}{G_L^2 + B^2} \right)\left(\frac{(G_L^2 + B^2) - 2B^2}{(G_L^2 + B^2)^2} \right) = 0$$

by simplification , we arrive at :

$$2X_T B^2 - 2(1 + 2G_L R_T)B - 2X_T G_L^2 = 0 \tag{3.33}$$

This gives us the optimal B denoted by B^* , where

$$B^* = \frac{2(1 + 2G_L R_T)}{2(2X_T)} \pm \sqrt{\left\{ \left(\frac{1 + 2R_T G_L}{2X_T} \right)^2 + \frac{16X_T^2 G_L^2}{16X_T^2} \right\}}$$

$$B^* = \frac{1 + 2G_L R_T}{2X_T} \pm \sqrt{\left(\frac{1 + 2R_T G_L}{2X_T} \right)^2 + G_T^2} \tag{3.34}$$

From equation (3.30) $B^* = 2\pi f C_o - B_L$

this results in

$$C_o = \frac{(B^* + B_L)}{2\pi f} \tag{3.35}$$

Comparing (3.35) and (3.26) , it is seen that when the source impedance is considered, maximising the power factor or maximising the transmission efficiency is not the same as minimising the line current even if the source is sinusoidal .

Different system data were used to run a computer program using the equations obtained so far and the values of optimal capacitors were found. These results are given at the end of chapter four and are compared with the values obtained when the system is excited by a nonsinusoidal source.

CHAPTER FOUR

SHUNT COMPENSATION OF LINEAR LOADS WITH NONSINUSOIDAL SOURCE

4.0 Introduction

In this chapter. voltage distortions consisting of harmonic components leading to a power source being periodic but nonsinusoidal are taken into consideration with loads assummed to be linear. The procedure followed in chapter three is repeated here but the problem is more complicated. The problem of shunt compensation is formulated as a nonlinear problem. This chapter therefore resorts to using nonlinear optimisation techniques . The Golden line section approach , due to its simplicity and the few number of steps it requires to converge . makes it an acceptable candidate for solving the problem of shunt compensation.

4.1 Zero Source Impedance with Nonsinusoidal Source

Let $v_s(t)$ be the instantaneous periodic nonsinusoidal source voltage with harmonic values written as $v_{sk}(t)$ where k is the harmonic number in figure 4.1 . $v_s(t)$ is absolutely integrable and can therefore be resolved into the trigononometrical series known as the Fourier series.

$$v_s(t) = V_{dc} + \sqrt{2}V_{s1}\sin(\omega t+\alpha_1) + \sqrt{2}V_{s2}\sin(2\omega t+\alpha_2) + \ldots\ldots + \sqrt{2}V_{sk}\sin(k\omega t+\alpha_k) \tag{4.1}$$

Equation (4.1) may be further generalised by the assumption

$$\left.\begin{aligned} V_o &= \frac{V_{dc}}{\sqrt{2}} \\ \alpha_o &= \frac{\pi}{2} \\ \omega_o &= 0 \end{aligned}\right\} \tag{4.2}$$

$V_s(t)$ can be written using (4.2) as :

$$V_{dc} = \sqrt{2}V_0 \sin(\omega_o t + \alpha_o) \tag{4.3}$$

$$v_s(t) = \sqrt{2}\sum_{0}^{k} V_{sk} \sin(k\omega t + \alpha_k)$$

(4.4)

or simply

$$v_s(t) = \sum_{k} V_{sk}(t) = \sqrt{2}\sum_{k} V_{sk} \sin(k\omega t + \alpha_k) \tag{4.5}$$

Using (4.5) the source current $i_s(t)$ can be written as

$$i_s(t) = \sum_{n} i_{sk}(t) = \sqrt{2}\sum_{k} I_{sk} \sin(k\omega t - \theta_k) \tag{4.6a}$$

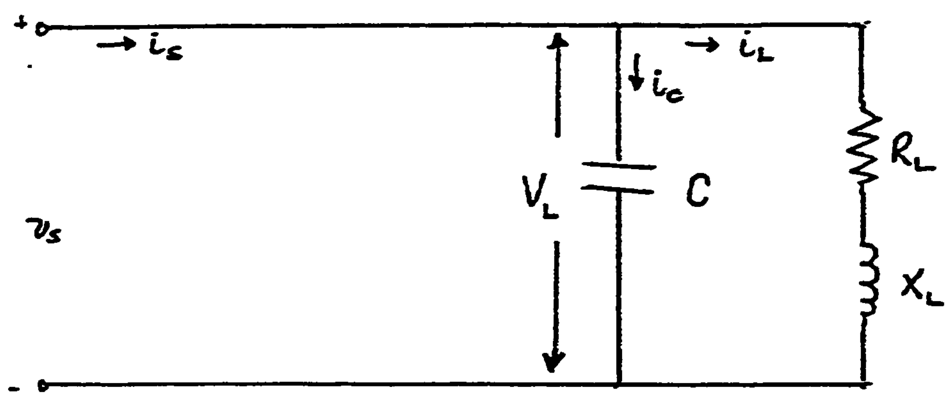


Figure 4.1 Capacitor compensation of zero source impedance with nonsinusoidal source and linear load

However . in terms of (4.4) we can rewrite (4.6a) as :

$$i_s(t) = \sqrt{2}\sum_n I_{sn}\sin(n\omega t + \alpha_n - \theta_n) \qquad (4.6b)$$

n is the number of the current harmonics. When the load impedance is linear the only current harmonics that will flow must correspond to the supply voltage harmonics. Hence . for linear circuits n=k and (4.6) can be written as :

$$i_s(t) = \sum_k i_{sk}(t) = \sqrt{2}\sum_k I_{sk}\sin(k\omega t + \alpha_k - \theta_k) \qquad (4.7)$$

where

$$I_{sk} = \frac{V_{sk}}{|Z_{Lk}|} \qquad (4.8)$$

$$Z_{Lk} = |Z_{Lk}| \,/\, \theta_k \qquad (4.9)$$

$$|Z_{Lk}| = \sqrt{R_{Lk}^2 + X_{Lk}^2} \qquad (4.10)$$

$$R_{Lk} = R_L \quad \textit{fundamental value} \qquad (4.11)$$

$$X_{Lk} = kX_L \qquad (4.12)$$

ω_o = fundamental frequency of the system (4.13)

θ_k = angle between the voltage and the current at the K-th harmonic. For this special case (zero source impedance)

$$\theta_k = \tan^{-1}\frac{X_{Lk}}{R_{Lk}} \qquad (4.14)$$

$$Y_{Lk} = G_{Lk} - jB_{Lk} \quad \text{linear load impedance} \qquad (4.15)$$

$$G_L(1) = \textit{fundamental conductance value} = \frac{R_L}{R^2_L + X^2_L} \qquad (4.16)$$

$$B_L(1) = \textit{fundamental susceptance value} = \frac{X_L}{R^2_L + X^2_L} \qquad (4.17)$$

$$G_{Lk} = \frac{R_L}{R^2_L + (kX_L)^2} \tag{4.18}$$

$$B_{Lk} = \frac{kX_L}{R^2_L + (kX_L)^2} \tag{4.19}$$

4.1.2 RMS relationships

The rms value of $v_s(t)$ is defined as :

$$V^2_s = \frac{1}{T}\int_0^T v^2_s(t)dt \tag{4.20}$$

In terms of a summation of Fourier harmonic components the rms values of the voltage $v_s(t)$ and the current $i_s(t)$ are :

$$V^2_s = V^2_{os} + \sum_0^k V^2_{sk} \tag{4.21}$$

$$I^2_s = I^2_{os} + \sum_0^k I^2_{sk} \tag{4.22}$$

[3]

where V_{sk} and I_{sk} are the rms voltage or current at the harmonic number k . Where the periodic function has zero time-average value, the rms voltage and current reduces to :

$$V^2_s = \sum_1^k V^2_{sk} \tag{4.23}$$

and

$$I^2_s = \sum_1^k I^2_{sk} \tag{4.24}$$

4.1.3 Energy and power relationships

The instantanous rate of energy flow into the linear circuit of figure 4.1 is given by

$$p = v_s(t)i_s(t) \tag{4.25}$$

The net energy W flowing into the circuit in time T is

$$W = \int_0^T v_s(t)i_s(t)dt = \int_0^T p dt \tag{4.26}$$

The average rate of energy flow (average power) P is

$$P = \frac{W}{T} \quad \text{which gives}$$

$$P = \sum_1^k V_{sk}I_{sk}\cos\theta_k \tag{4.27}$$

If we replace θ_k *by* φ_k the usual symbol for the power factor, then φ_k is the angle between the k-th harmonic voltage and current components. From the principle of conservation of energy . the validity of (4.27) can easily be proved. A non zero time-average power can only be transferred by the combination of voltage and current components of the same frequencies. Hence P cannot contain terms like $V_{sk}I_{sn}$ *unless* $n=k$. In a similar way the reactive voltamperes Q is expressed as :

$$Q = \sum_k V_{sk}I_{sk}\sin\varphi_k \tag{4.28}$$

The apparent voltamperes S is expressed as

$$S^2_s = V^2_{RMS}I^2_{RMS}$$

$$= \sum_k V^2_{sk}\sum_k I^2_{sk} \tag{4.29}$$

(4.29) is a figure of merit defining the maximum energy transfer capability of the system as discussed in chapter two. The active voltampers S_R can be deduced from (4.29) as

$$S^2_R = \sum_k V^2_{sk} \sum_k (I_{sk}\cos\varphi_k)^2 \quad (4.30)$$

The energy storage which is the reactive voltamperes S^2_{LC} *is given by*

$$S^2_{LC} = \sum_k V^2_{sk} \sum_k (I_{sk}\sin\varphi_k)^2 \quad (4.31)$$

The distortion voltamperes D is therefore

$$D^2 = S^2 - S^2_R - S^2_{LL} \quad (4.32)$$

These are the equations of Budeanu and Kusier-Moore discussed in chapter two. These equations are now applied in solving the problem of shunt compensation at nonsinusoidal busbars .

4.2 Mathematical formulation of the optimization problem

Let

$$G_L = \sum_{k=1}^{k} G_{Lk} \quad \text{and} \quad B_L = \sum_{k=1}^{k} B_{Lk} \quad (4.33)$$

The load current for the k-th harmonic I_{Lk} *is defined as*

$$I_{Lk} = (G_{Lk} - jB_{Lk})V_{Lk} \quad (4.34)$$

Where V_{Lk} is the load k-th harmonic voltage . For zero source impedance

$$V_{Lk} = V_{sk}$$

The capacitor current for the k-th harmonic is :

$$I_{ck} = jk\omega_o CV_{Lk} \quad (4.35)$$

The rms source current for the K-th harmonic is :

$$I_{sk} = I_{Lk} + I_{ck}$$

$$= (G_{Lk} + j(k\omega_o C - B_{Lk}))V_{Lk} \quad (4.36)$$

$$I_s^2 = \sum I^2_{sk} \quad (4.37)$$

$$= \sum (G^2_{Lk} + (k\omega_o C - B_{Lk})^2) V^2_{Lk} \qquad (4.38)$$

Since $V_{Lk} = V_{sk}$

$$I_s^2 = \sum (G^2_{Lk} + (k\omega_o C - B_{Lk})^2) V^2_{sk} \qquad (4.39)$$

4.2.1 Minimising the Line Current (or line losses I^2_s)

For minimising the line current or line losses. Equation (4.39) is differentiated with respect to C and the result is equated to zero .

$$\frac{dI^2_s(C)}{dC} = 0 = 2\omega_o \sum k V^2_{sk}(k\omega_o C - B_{Lk})$$

$$C_o = \frac{\sum k B_{Lk} V^2_{Lk}}{\omega_o \sum k^2 V^2_{Lk}} \qquad (4.40)$$

This . as seen . is quite different from (3.20) which was the expression obtained for minimising the line current (or losses) under zero source impedance with sinusoidal source and linear load .

4.2.2 Maximising the Power Factor (PF)

The compensated power factor at the load is given by ;

$$PF_L = \frac{P_L}{V_L I_s} \qquad (4.41)$$

Where

$$P_L = \sum V^2_{Lk} G_{Lk} \qquad (4.42)$$

Combining equations (4.41), (4.42), (4.23) , and (4.24) we get :

$$PF(C) = \frac{\sum V^2_{Lk} G_{Lk}}{\sqrt{\sum V^2_{LK} \sum I^2_{sk}}} \qquad (4.43)$$

from (4.43) we realise that $\sum V^2_{Lk}$ and G_{Lk} are independent of the compensating capacitor and are therefore constants. Hence to maximise the power factor from (4.32) we should minimise the line current or line losses $\sum I^2_{sk}$. This is the same as minimising the line current as carried out before . We therefore conclude that for zero source impedance with nonsinusoidal voltage and a linear load . maximising the power factor or minimising the line current (or losses) yield the same optimal compensating capacitor given by :

$$C_o = \frac{\sum k B_{Lk} V^2_{Lk}}{\omega_o \sum k^2 V^2_{Lk}} \tag{4.40}$$

4.2.3 Maximising the Transmission efficiency η

We need to compute the capacitance C such that

$$\eta = \frac{P_L}{P_s} \tag{4.44}$$

is a maximum. Where

$$P_L = \sum G_{Lk} V^2_{Lk}$$

as in (4.42) above and

$$P_s = P_L + \sum R_{Tk} I^2_{sk} \tag{4.45}$$

Where R_{Tk} is the transmission line resistance at harmonic number k . Since we are considering zero source impedance $R_{Tk} = 0$, then it is obvious that the criterion leading to maximum transmission efficiency does not make any sense here.

The conclusion reached here is that , for zero source impedance with nonsinusoidal supply and a linear load , the optimal shunt capacitor for compensation is obtained by considering only maximising the power factor and minimising the line current . Both two criteria leads to the same value of optimal capacitor given by (4.40)

4.3 Non Zero Source Impedance with Nonsinusoidal source and Linear Load Impedance

In figure 4.2 . R_{Tk} and X_{Tk} are the source k-th harmonic resistance and reactance respectively. The fundamental values are denoted simply as R_T and X_T .

$$R_{Tk} = R_T$$

$$X_{Tk} = kX_T$$

$Z_{Tk} = R_{Tk} + jX_{Tk}$ Transmission line K-th harmonic impedance .

$$= R_T + jkX_T \qquad (4.46)$$

Similarly. the load impedance at the k-th harmonic is

$$Z_{Lk} = R_{Lk} + jX_{Lk}$$

$$= R_L + kX_L \qquad (4.47a)$$

The impedance of the shunt capacitor is given by :

$$Z_{ck} = X_{ck} = \frac{1}{jk\omega_o C}$$

$$B_{ck} = \frac{1}{X_{ck}}$$

$$B_{ck} = jk\omega_o C = Y_{ck} \qquad (4.48)$$

$$Y_{Tk} = G_{Tk} - jB_{Tk} \qquad (4.49)$$

$$Y_{Lk} = G_{Lk} - jB_{Lk} \qquad (4.50)$$

The load and source resistance are assumed to remain at their fundamental value i.e. the skin effect is neglected at the harmonic frequencies.

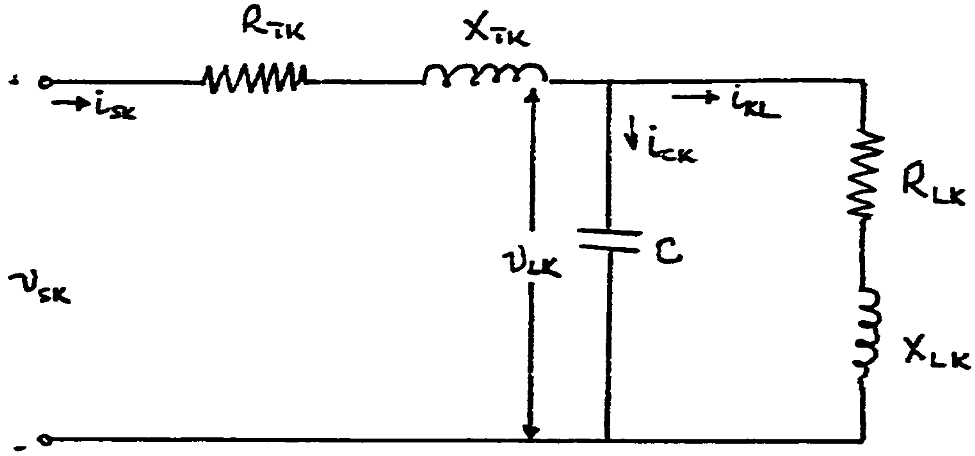


Figure 4.2 Non zero source impedance,nonsinusoidal source

Let $Y_{cL} = Y_{ck}$ *in parallel with* Y_{Lk} .

$$Y_{cL} = Y_{Lk} + Y_{ck}$$

$$= G_{Lk} + j(k\omega_o C_o - B_{Lk}) \tag{4.51}$$

The total impedance Z of the circuit of figure 4.2 is

$Z = Z_{Tk}$ *in series with* Z_{cL}

Where $Z_{cL} = \dfrac{1}{Y_{cL}}$

$$Z = Z_{Tk} + Z_{cL}$$

$$= Z_{Tk} + \frac{1}{Y_{cL}}$$

$$= \frac{1 + Z_{Tk}Y_{cL}}{Y_{cL}} \tag{4.52}$$

From the voltage division rule , the load voltage V_{Lk} *is* :

$$V_{Lk} = \frac{V_{sk}Z_{cL}}{Z}$$

$$= \frac{V_{sk}}{1 + Z_{Tk}Y_{cL}} \tag{4.53a}$$

or

$$V_{Lk} = \frac{V_{sk}}{1 + Z_{Tk}(G_{Lk} + j(k\omega_o C - B_{Lk})\ )} \tag{4.53b}$$

From (4.38) , the square of the line current is

$$I^2_s = \sum\left\{G^2_{Lk} + (k\omega_o C - B_{Lk})^2\right\}V^2_{Lk} \quad or$$

$$= \sum(G^2_{Lk} + (k\omega_o C - B_{Lk})^2)\left(\frac{V_{sk}}{1+Z_{Tk}(G_{Lk}+j(k\omega_o C-B_{Lk})\ )} \right)^2$$

(4.54)

Substituting (4.46) for Z_{T_k} into (4.54) and grouping real and imaginary terms, we obtain :

$$I_s^2 = \sum \frac{G^2_{Lk} + (k\omega_o C - B_{Lk})^2}{K^2_1 + K^2_2} V^2_{sk} \tag{4.55}$$

Where

$$\left.\begin{aligned} K_1 &= 1 + R_T G_{Lk} - kX_T(k\omega_o C - B_{Lk}) \\ &\text{and} \\ K_2 &= kX_T G_{Lk} + R_T(k\omega_o C - B_{Lk}) \end{aligned}\right\} \tag{4.56}$$

Since the source impedance is considered . one condition to guard against is that of resonance as was mentioned in chapter one. The resonance condition will occur if the imaginary part of the combined total impedance Z of the system is zero . Therefore (4.52) must be rearranged into real and imaginary parts. The resonance that might occur at any given harmonic number k can lead to large oscillating harmonic currents over loading circuit elements and causing failure or maloperation of protective equipment. To impose this constraint on the formulation of the optimisation problem, the capacitor values resulting in resonant phenomena have to be identified and eleminated from the solution process.

$$Z = R_T + jkX_T + \frac{(R_L + jkX_L)}{1 + jk\omega_o C(R_L + jkX_L)} \tag{4.57}$$

$$= R_T + jkX_T + \frac{R_L + jkX_L}{(1 - k^2\omega_o X_L C) + jk\omega_o R_L C}$$

$$= R_T + jkX_T + \frac{(R_L + jkX_L)((1 - k^2\omega_o C X_L) - jk\omega_o R_L C)}{(1 - k^2\omega_o X_L C)^2 + (k\omega_o R_L C)^2}$$

Equating the imaginary part to zero ;

$$kX_T + \frac{kX_L(1 - k^2\omega_o C X_L) - k\omega_o R^2_L C}{(1 - k^2\omega_o C X_L)^2 + (k\omega_o R_L C)^2} = 0$$

$$X_T\left((k^2\omega_o X_L)^2 + (K\omega_o R_L)^2 \right) C^2 - \omega_o(R^2_L + (kX_L)^2 + 2k^2X_LX_T)\, C + (X_T + X_L) = 0$$

(4.58)

Equation (4.58) is a quadratic equation in C and can therefore be written as :

$$A_1C^2 + A_2C + A_3 = 0 \qquad (4.59)$$

Where

$$A_1 = X_T\left((k^2\omega_o X_L)^2 + (k\omega_o R_L)^2 \right)$$

$$A_2 = -\omega_o\left(R^2_L + (kX_L)^2 + 2k^2X_LX_T \right)$$

and

$$A_3 = X_T + X_L$$

The solution of (4.59) is in the form :

$$C = \frac{-A_2 \pm \sqrt{A_2^2 - 4A_1A_3}}{2A_1} \qquad (4.60a)$$

or

$$\left.\begin{aligned} C_1 &= \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1} \\ &\text{and} \\ C_2 &= \frac{-A_2 - \sqrt{A_2^2 - 4A_1A_3}}{2A_1} \end{aligned}\right\} \qquad (4.60b)$$

The capacitor values C_i where i= 1, 2 are those which would create a series resonant and a parallel resonant conditions. When $X_T = 0$ in (4.59), we obtain C_3 which is the parallel resonant condition at the load .

$$C_3 = \left. \frac{-A_3}{A_2} \right|_{X_T=0}$$

$$C_3 = \frac{X_L}{\omega_o (R^2_L - k^2 X^2_L)} \tag{4.61}$$

However, a parallel resonant circuit at the load implies unity power factor. Hence , only the series resonant conditions which would result in large harmonic line currents are omitted from the solution.

A computer program is written to calculate C_1 . C_2 , and C_3. The values of C_1 and C_2 which are equal to C_3 are eliminated and the rest of the values of C_1 and C_2 are taken as constraints in the solution procedure.

4.3 Non-Zero Source Impedance With Nonsinusoidal Source and Linear Load Impedance

4.3.1 Mathematical Formulation Of The Optimization Problem

The choice of compensating capacitor is now formulated as optimization problem. The optimisation problem will handle each criteria separately. This is in line of refernce 4's work.

4.3.2 Minimising the line losses (I^2_s)

Tthe objective function I^2_s (C) which is to be to optimized (minimised) subject to the constraints given by (4.60) is formulated as shown below .The values of C_1 and C_2 in equation (4.60) are denoted by C_{ik} . The optimal capacitor value C_{opt} is found from the optimisation problem ;

$$\left. \begin{aligned} &\textit{minimise} \quad I_s^2(C) = \sum I_{sk}^2(C) \\ &\textit{subject} \quad C \neq C_{ik} \end{aligned} \right\} \tag{4.62}$$

Note that C_{ik} satisfies (4.60) but not (4.61) . I^2_s was found in (4.55) and written here as the objective function .

$$I_s^2(C) = \sum \frac{G^2_{Lk} + (k\omega_o C - B_{Lk})^2}{K^2_1 + K^2_2} V^2_{sk} \tag{4.63}$$

Where

$$\left.\begin{aligned} K_1 &= 1 + R_T G_{Lk} - kX_T(k\omega_o C - B_{Lk}) \\ &\text{and} \\ K_2 &= kX_T G_{Lk} + R_T(k\omega_o C - B_{Lk}) \end{aligned}\right\} \tag{4.64}$$

Equation (4.63) is not easily differentiable . The golden section search algorithm has been used here as the solution technique to minimise the single variable function of (4.63). The formulation of the search algorithm to solve (4.62) is presented later in this chapter .

4.3.3 Maximising the Power Factor (PF)

$$P_L = \sum G_{Lk} V^2_{Lk} \tag{4.65}$$

$$V^2_L = \sum V^2_{Lk} \tag{4.66}$$

$$V^2_L = \sum \frac{V^2_{sk}}{K^2_1 + K^2_2} \tag{4.67}$$

$$PF = \frac{\sum G_{Lk} V^2_{Lk}}{\sqrt{V^2_L I^2_s}} \tag{4.68}$$

(4.68) is the objective function . The optimal C value is found from the following optimisation problem;

$$\left.\begin{aligned} &\text{maximise} \quad PF(C) = \frac{P_L(C)}{V_L I_s(C)} \\ &\text{subject} \quad : \quad C \neq C_{ik} \end{aligned}\right\} \tag{4.69}$$

This problem was also solved using the Golden section method [4]. The non unimodal nature of the curves of equations (4.62) and (4.69) are shown on figure 4.3 for eample 1 of section 4.4.3

Nature of curves

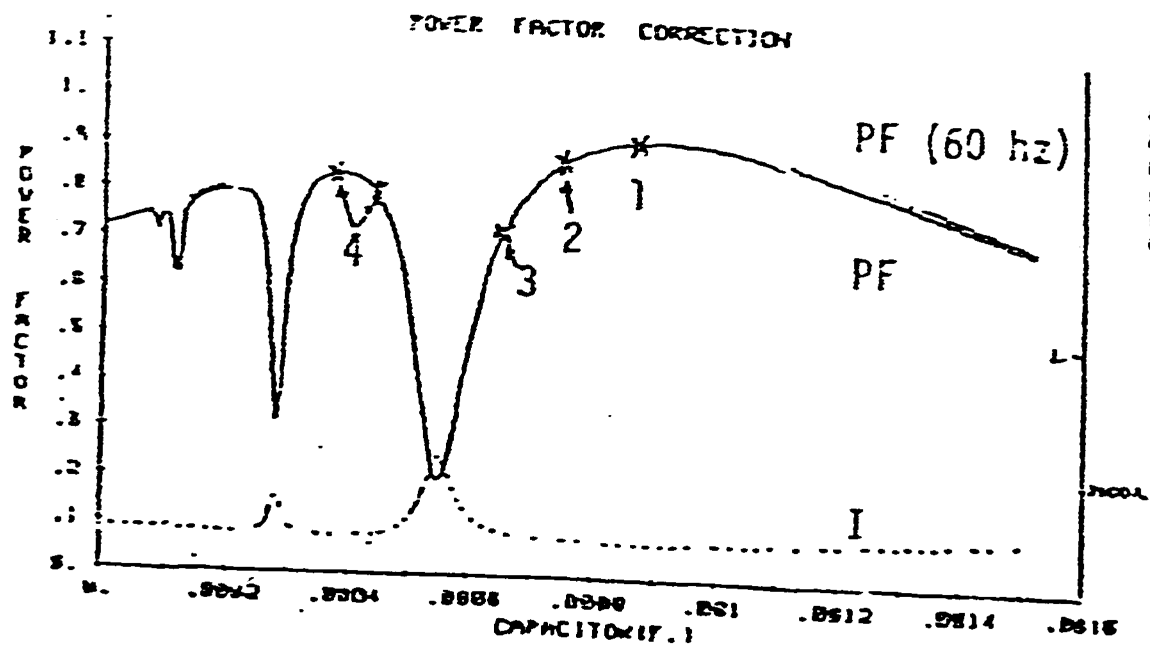


Figure 4.3 Plots of the Line Current and Power Factor for Example 1 (80 MVA)

4.3.4 Maximising the Transmission Efficiency (η)

Using the transmission efficiency as a criteria. the optimal C value can be found from the following optimisation problem.

$$\left.\begin{array}{ll} \textit{maximise} & \eta = \dfrac{P_L(C)}{P_s(C)} \\ \textit{subject} : & C = C_{ik} \end{array}\right\} \tag{4.70}$$

Where

$$P_s = P_L + \sum R_{Tk} I_{sk}^2$$

$$= \sum G_{Lk} V_{LK}^2 + {}_{I_{sk}}^{2} R_{Tk} \tag{4.71}$$

$$\eta(C) = \frac{\sum G_{LK} V_{Lk}}{\sum G_{Lk} V_{Lk}^2 + \sum I_{sk}^2 R_{Tk}} \tag{4.72}$$

Equation (4.72) is the objective function of (4.70) which was also solved using the Golden section search algorithm .

4.4 Formulation of the Search Algorithm

The objective functions for the three criteria under consideration are defined by (4.62), (4.69). and (4.70). Various direct methods can be applied but that of the Golden Section Search algorithm is employed here due to the simplicity it offers in requiring fewer steps and function evaluations.

4.4.1 Golden Section Search Algorithm

The Golden section search algorithm is outlined below [4 & 26].

Let

C_u - upper bound of the search interval of capacitor value

C_L - lower bound of the search interval of capacitor value

I - interval at each iteration step

C_1 , C_2 - points within the interval where $C_1 < C_2$

f(c) - objective function and

ε - convergence criterion for the algorithm (ie the smallest possible interval of uncertainty in which the minimum lies) .

lamda is a parameter of the Golden section search algorithm defined as ;

$$\lambda = \frac{(3-\sqrt{5})}{2} = 0.381966$$

Figure 4.4 illustrate the problem

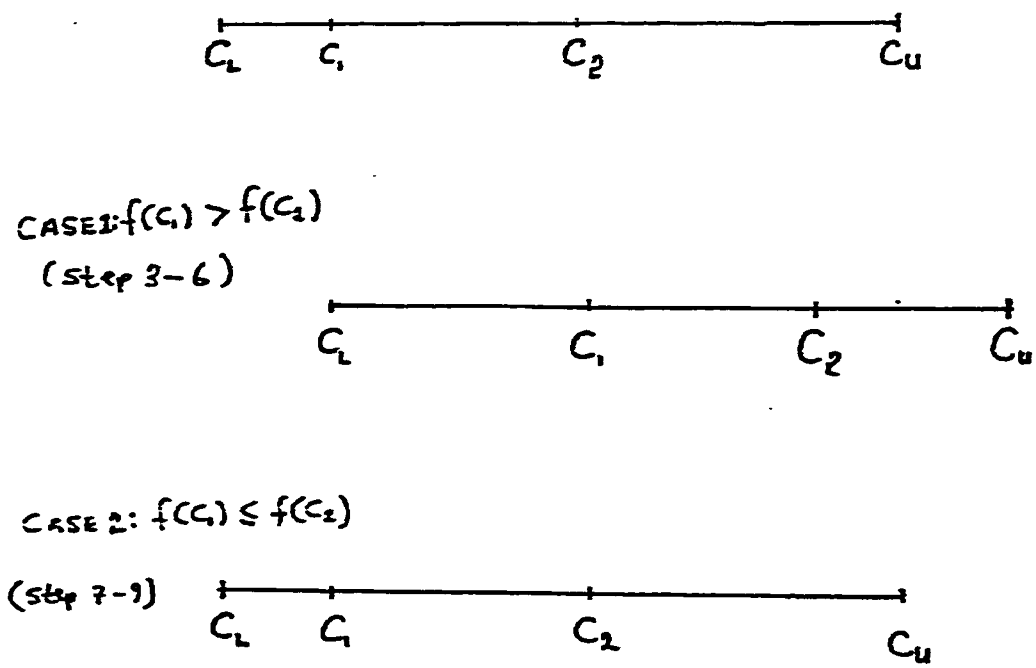


Figure 4.4 Illustrating golden section search method

The following steps illustrate the algorithm.

Step 1 :

Calculate $I = C_u - C_L$

$C_1 = C_L + \lambda I$

and $C_2 = C_u - \lambda I$

Evaluate $f(C_1)$ and $f(C_2)$

Step 2 :

If $f(C_1) \leq f(C_2)$. go to step 7

Step 3 :

else if $f(C_1) > f(C_2)$

set $C_L = C_1$ and $f(C_L) = f(C_1)$

Step 4 :

set $C_1 = C_2$ and $f(C_1) = f(C_2)$

Step 5 :

set $C_2 = C_u - \lambda(C_u - C_L)$

and *evaluate* $f(C_2)$

Step 6 :

go to 10

Step 7 :

set $C_u = C_2$ and $f(C_u) = f(C_2)$

Step 8 :

set $C_2 = C_1$ and $f(C_2) = f(C_1)$

Step 9 :

set $C_1 = C_L + \lambda(C_u - C_L)$

and *evaluate* $f(C_1)$

Step 10 :

If $(C_u - C_L) \geq \varepsilon$, go to step 2 otherwise stop.

4.4.2 Flow Chart

The precalculated capacitor values for series resonance C_{ik} are used to subdivide the entire search region into numerical small regions. This is to avoid convergence to the several local minimams possible as a result of the resonance conditions . Within these regions . the local minimams are identified and hence . the global minimum .

The lower bound of the capacitor value C_L for initiating the search algorithm is zero . As for the upper bound C_u . the employed value is the one which could have produced a leading power factor (based on fundamental frequency analyses) equal to the uncompensated initial power factor . The flowchart is shown on figure 4.5 overleaf.

Flow - chart for the developed algorithm

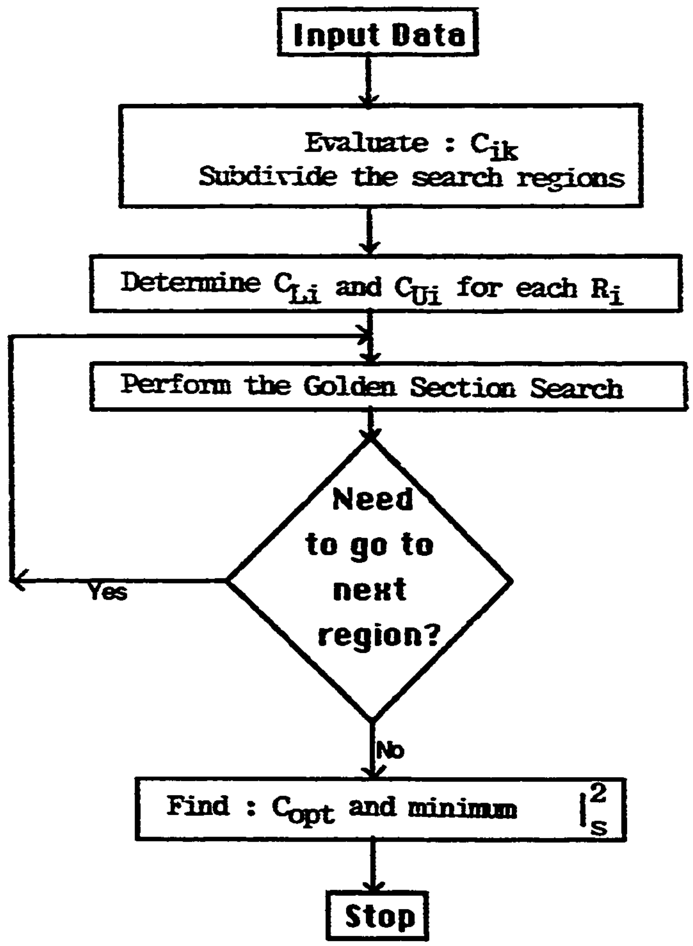


Table 4.1 : Summary of equations on the operation of a linear load

Line condition	Sinusoidal voltage	Nonsinusoidal voltage
Zero source impedance	$C_o = \frac{B_L}{2\pi F}$ which **realises both unity power factor and minimum line current**	$C_o = \frac{\sum k B_{Lk} V_{Lk}^2}{\omega_o \sum k^2 V_{Lk}^2}$ which **realises both maximum power factor and minium line current**
Non zero source impedance	**(1) Maximising the power factor and efficiency** $C_o = \frac{B_L}{2\pi F}$ **(2) Minimising the line current** $B^* = \frac{1+2G_L R_T}{2X_T} \pm \sqrt{\{\frac{1+2G_L R_T}{2X_T}\}^2 + G_L^2}$ $C_o = \frac{B_L + B^*}{2\pi F}$ **This is denoted as C_s**	**Numerical optimisation** **(1) Minimising the line losses (current)** compute C_o when $I_s^2 = \sum I_{sk}^2$ **is minimised** **(2) Maximising the power factor PF :** $PF = \frac{P_L}{V_L I_s}$ $P_L = \sum V_{Lk}^2 G_{Lk}$ $I_s^2 = \sum I_{sk}^2$, $V_L^2 = \sum V_{Lk}^2$ Compute C_o when $PF = \frac{\sum V_{Lk}^2 G_{Lk}}{\sqrt{I_s^2 V_L^2}}$ **is a maximum** **(3) maximising the transmission efficiency :** $\eta^* = \frac{P_L}{P_s}$

4.4.3 Examples

Example 1 :

A three phase load of 5100 KW and 4965 KVAR is connected to a supply bus with voltage 4160 V (2400 line-to-ground). 60 Hz frequency and 80 MVA short circuit capacity. The resistance to reactance ratio of the power system impedance is assumed to be 10 % . The voltage is distorted : it contains 5 % fifth harmonic. 3 % seventh harmonic. 2 % eleventh harmonic. and 1 % thirteenth harmonic. The system data for an equivalent single phase mode are :

power system resistance : 0.02163Ω

power system reactance : 0.2163Ω

load resistance : 1.7421Ω

load reactance : 1.696Ω

frequency : 60 Hz

Voltage :

fundamental component : 2400 V

5 th harmonic : 120 V

7 th harmonic : 72 V

11 th harmonic : 48 V

13 th harmonic : 24 V

The results of this example from a computer program using the equations obtained so far are summarized in table 4.2

Table 4.2 : Summary of results for example 1

Condition of circuit	Objective	Capacitor mF	η - %	I_s - A	PF
zero source impedance with sinusoidal source	*Maximising* the η - and PF -	*0.761*	*99.18*	*795.45*	*0.8767*
Non zero source impedance with sinusoidal source	Minimising I_s^2 -	0.712	99.10	832.45	0.8311
zero source impedance with non-sinusoidal source	Minimising I_s^2 - and max. PF -	0.654	98.86	914.57	0.725
Non zero source impedance with Non-sinusoidal source	Maximising only the PF	0.910	99.25	771.88	0.9179
Non zero source impedance with Non-sinusoidal source	Minimising only I_s^2 -	0.880	99.25	770.77	0.9166
Non zero source impedance with Non-sinusoidal source	Maximising the η -	0.906	99.25	771.57	0.9178

Example 2

Consider the following data :

$R_T = 0.1\Omega$. $X_T = 0.2\Omega$. $R_L = 1.0\Omega$, and $X_L = 3\Omega$

frequancy : 50 Hz

Voltage harmonics :

fundamental voltage : 100 V

5 th harmonic : 10 % of fundamental voltage

7 th harmonic : 10 % of fundamental voltage

The results of this example are tabulated in table 4.2

In these examples the skind effect is neglected. For a system in which the factor of skind effect is known it can be used to find the variation of the line resistance with harmonic components. In addition , the voltage levels are assumed constant. The variations of busbar voltages can be taken into account by runing a dynamic load program on the system before and after compensation.

Table 4.3 : Summary of results for example 2

Condition of circuit	Objective	Capacitor mF	η - %	I_s - A	PF
zero source impedance with sinusoidal source	Maximising the η - and PF -	0.955	89.17	34.65	0.283
Non zero source impedance with sinusoidal source	Minimising I_s^2 -	0.949	88.90	35.00	0.276
zero source impedance with non-sinusoidal source	Minimising I_s^2 - and max. PF -	0.561	81.70	46.04	0.189
Non zero source impedance with Non-sinusoidal source	Maximising only the PF	1.310	93.54	26.60	0.3799
Non zero source impedance with Non-sinusoidal source	Minimising only I_s^2 -	0.181	93.01	25.91	0.354
Non zero source impedance with Non-sinusoidal source	Maximising the η -	1.306	93.54	26.58	0.379

From the two tables , it is realised that different optimum capacitor values for different conditions of operation are obtained . The sensitivety of the developed approach was tested by varying the harmonic component values within the *limits of* $\pm$ 10 *per cent*. The results were the same as those obtained in the two tables. The second example gives a very low operating power factors the maximum of which is 0.37. Practically, this is not an accepted operating power factor and the presence of a compensating capacitor has not improved the power factor to an acceptable limit of operation which is within 0.8 to 1.0 . A suggested solution procedure introduced in this work is the Penalty function approach which is the subject of discussion in the next section presented below .

4.5 Using Penalty Function

By the use of penalty function, the problem of minimisimg the line losses with the expressions for the power factor and the efficiency taken as constraints can be reformulated. To do this the penalty function approach in inequality constrained problem is utilized.[28]

4.5.1 Inequality Constraint

$$\left.\begin{array}{ll} \textit{minimise} & f(x) \\ \textit{subject} & g(x) \le 0 \\ & X \in E_n \end{array}\right\} \qquad (4.73)$$

E_n is the set containing all possible values of X .

This problem is transformed to :

$$\min \quad f(x) + \mu\,(\max\{0, g(x)\}) \qquad (4.74)$$

subject $\quad x \in E_n$

If $g(x) \le 0$, then max [0 , g(x)] = 0 , and there is no penalty . If

$g(x)>0$, *then* $\max\{0, g(x)\}>0$. and the penalty term $\mu g(x)$ is realised .

4.5.2 Algorithm

Initialization step :

Let $\varepsilon>0$ be a termination scalar. Choose an initial point X_1 (the optimal solution to the original problem f(x)). a penalty parameter $\mu>0$ and *a scalar* $\beta>1$. Let k = 1 and go to the main step

main step :

1. Starting with X_k . solve the following problem :

minimise $f(x) + \mu_k(\max\{0, g(x)\})$ (4.75)

Subject $x \in E_n$

Let X_{k+1} be an optimal solution , and go to step 2 .

2. If $\mu_k(\max\{0, g(X_{k+1})\}) < \varepsilon$ stop: otherwise let $\mu_{k+1} = \beta\mu_k$, replace k by k + 1 and go to step 1 [15]

4.5.3 Application of the Penalty Function

The practical operation limits of the power factor is within 0.8 to 1.0 and that of the effeciency is within 0.85 to 1.0 . These limits are made use of in this section to constrain the power factor and the effeciency within acceptable practical operation limits. The objective function is formulated as shown below.

Minimise $I_s^2(C)$

subject $0.85 \le PF(C) \le 1.0$

$0.85 \le \eta(C) \le 1.0$

$C \ne C_{ik}$

Where the C_{ik}'s are the values of C that would cause resonance as discussed earlier on.

This problem can be rewritten in the form below . For simplicity the efficiency constraint is eliminated because it leads to high values of compensation capacitor values as shown in the examples. However, it should be made clear that in solving the problem , the efficiency was included later after solving the problem without it. This obviously gave the same solution since the values of the efficencies were above 0.9 .

Minimise $\quad I_s^2(C)$

subject $\quad 0.85 - PF(C) \leq 0.0$

$$PF(C) - 1.0 \leq 0.0$$

$$C \neq C_{ik}$$

Denoting $g_1(x)$ *by* $0.85 - PF(C)$ and $g_2(x)$ *by* $PF(C) - 1.0$, the problem can be rewritten as :

Minimise $\quad I_s^2(C)$

subject $\quad g_1(x) \leq 0.0$

$$g_2(x) \leq 0.0$$

$$C \neq C_{ik}$$

The above can be expressed as a single objective function in the form:

$$\text{minimise} \quad I_s^2(C) + \lambda_1\{\max(0, g_1(x))\} + \lambda_2\{\max(0, g_2(x))\} \tag{4.76}$$

subject $\quad C \neq C_{ik}$

Where C_{ik}'s are the resonant values and Lambdas are chosen arbitrarily.

Using the algorithm in section 4.5.2 with examples one and two . the following results are obtained :

For example one -

minimum line current $I_s = 771.625A$

power factor PF = 0.9133

efficency $\eta = 0.9925$

The value of the optimum capacitor $C_o = 0.854mF$

These results are the same optimum values obtained earlier . This is possible because for this example the power system could yield a power factor within the range of 0.85 to 1.0 .

For example two -

The desired solution to obtain a power factor higher than the 0.37 of the system data was not possible within the range of capacitors values chosen . However , it should be added that upon persuing to solve the problem outside that range , it was realised that no practical value of capacitor could provide the desired solution. It is therefore concluded that for a particular system data , the power factor can not be improved beyond a specific value . This is so because the power factor is dependent on the system data . In trying this procedure within the range of capacitor values specified , the following results were obtained.

$I_s = 26.59A$

power factor PF = 0.378

efficency $\eta = 0.9353$

The value of the optimam capacitor $C_o = 1.302mF$

These values are similar to the optimam values obtained earlier . In the next chapter nonlinear loads under nonsinusoidal conditions is discussed.

CHAPTER FIVE

CONSIDERATION OF NONLINEAR LOADS WITH NONSINUSOIDAL SOURCE

5.1 Introduction

When a nonsinusoidal voltage is applied to a circuit consisting of at least one nonlinear load . the supply current becomes nonsinusoidal and may be of different waveform from the supply voltage. In the electric distribution network . such a situation might occur if a nonlinear load such as a rectifier installation exist at some distance from the generation point. The nonsinusoidal current drawn by this nonlinear load through the supply impedance leads to the distortion of the supply voltage both at the terminals of the load and the terminals of other. possibly linear loads on the same feeder [3] . This undesirable condition must be avoided . If the load is considered to be linear then compensation by shunt capacitor is sufficient to arrest abnormal situations. In the presence of nonlinear loads which result in the proliferation of the network with harmonic currents, the best solution is to use a tuned filter (inductive reactor) to take care of these harmonic currents. These filters are usually resorted to only in the case of heavily nonlinear loads because of their associated high cost. In this discussion, the load harmonics are assumed not to be sufficiently serious to require tuned filters but when combined with the nonsinusoidal source harmonics, the use of a pure capacitive compensator may degrade power factor and overload the equipment. As such , a solution is sought using combination of capacitors and inductors (LC)

5.2 uncompensated System

Figure 5.1 is a circuit of nonlinear load being supplied from a distorted

(nonsinusoidal) source voltage v_{sk}. The nonlinear load can be represented as a load with a current source I_{Lk} $(k > 1)$ injecting harmonic currents into the system.

$$V_s = \sum Vsk \tag{5.1}$$

Where V_s is the RMS supply voltage and V_{sk} is the k-th harmonic supply voltage.

$$Z_{tk} = R_{tk} + jX_{tk} \tag{5.2}$$

Z_{tk} is the transmission line impedance for the k-th harmonic

R_{tk} is the transmission line resistance for the k-th harmonic

and X_{tk} is the transmission line reactance for the k-th harmonic

V^o_{lk} is the load voltage for the k-th harmonic in the absence of compensation

I^o_{sk} is the supply RMS current of the k-th harmonic

I_{Lk} is the k-th current harmonic produced by the nonlinear load .

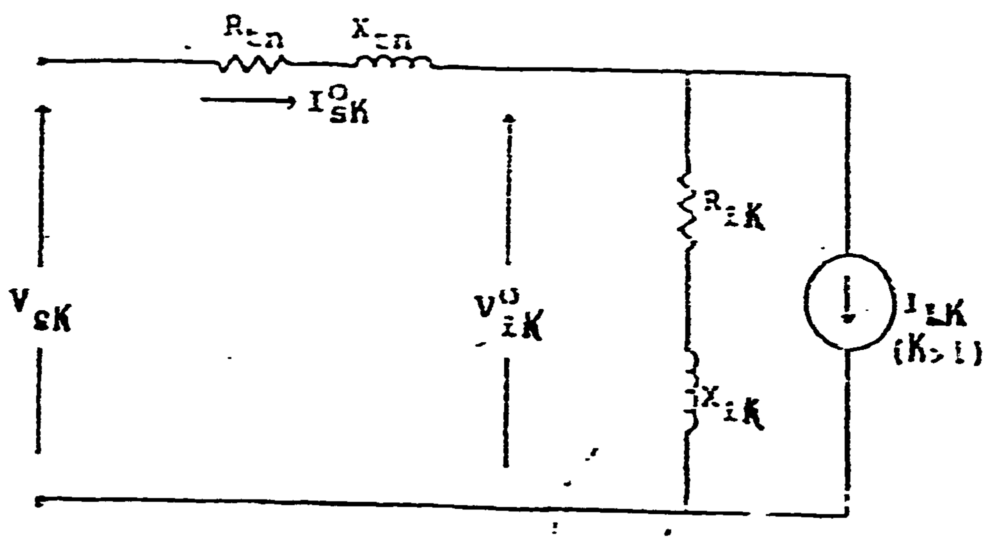


Figure 5. 1 Nonlinear load with nonsinusoidal supply (no compensation)

The load impedance is given by

$$Z_{lk} = R_{lk} + jX_{lk} \tag{5.3}$$

$$I^{o}_{L} = \sum_{k=1} i_{Lk} \tag{5.4}$$

$$V_{sk} = (R_{tk} + jX_{tk}) I^{o}_{sk} + V^{o}_{lk} \tag{5.5}$$

$$V^{o}_{lk} = (I^{o}_{sk} - I_{Lk})(R_{lk} + jX_{lk}) \tag{5.6}$$

Combining (5.5) and (5.6) we get :

$$I^{o}_{sk} = \frac{V_{sk} + I_{Lk}(R_{lk} + jX_{lk})}{(R_{tk} + R_{lk}) + j(X_{tk} + X_{lk})} \tag{5.7a}$$

$$I^{o}_{s} = \sum I^{o}_{sk} \tag{5.7b}$$

$$V^{o}_{L} = \sum V^{o}_{Lk} \tag{5.8}$$

From these equations we can find the power factor and the efficiency.

Power Factor (PF) :

$$PF = \frac{P_L}{I_s V^{o}_{L}} \tag{5.9}$$

$$P_L = G_L (V^{o}_{L})^2 \tag{5.10}$$

$$G_L = \sum G_{Lk} \tag{5.11}$$

$$G_{Lk} = \frac{R_{Lk}}{R^2_{Lk} + X^2_{Lk}} = \frac{R_L}{R^2_L + k^2 X^2_{L}} \tag{5.12}$$

$$PF = \frac{\sum G_{Lk} \sum V^{o2}_{Lk}}{\sum I^{o}_{sk} \sum V^{o}_{Lk}} \tag{5.13}$$

Efficiency (η) :

$$\eta = \frac{P_L}{P_s} \tag{5.14}$$

$$P_s = \sum R_{tk} I^{o2}_{sk} + P_L$$

$$= R_T I_s^{o2} + P_L \tag{5.15}$$

Where $R_T = R_{tk}$

$$\eta = \frac{\sum G_{Lk} \sum V_{Lk}^{o2}}{R_T \sum I_{sk}^{o2} + \sum G_{Lk} \sum V_{Lk}^{o2}} \tag{5.16}$$

Using the data of example 1 of chapter three and arbitary values of the nonlinear load harmonic currents as I_{Lk}= 0.0 . 0.0 . 304 . 0.0 . 33.0 . 0.0 . 25.0 . 0.0 . 26.0 . 0.0 . 8.0 . 0.0 . 9.0 . the following values were obtained :

Line current I_s^o = 963.593 A

Power factor PF = 0.684

and the transmission efficiency = 0.98

Using the data of example 2 of chapter three and arbitary values of I_{Lk}= 0.0 , 10.0 , 0.0 , 7.0 , 0.0 , 3.0 , 0.0 for the nonlinear load , the following values were obtained :

Line current I_s^o = 31.822 A

Power factor PF = 0.23

and the transmission efficiency = 0.89

Comparing these results with table 4.1 and table 4.2 for the same system data but with the load being linear , we realised a degraded power factor , a high line current, and hence an increase in line losses and a decrease in the transmission efficiency. For this reason, compensation will be inevitable in such situations and L-C shunt compensation is to be considered .

5.3 With Compensation

Using figure 5.2 with the subscripts having the same meanings as in figure 5.1 . X_c is the capacitive reactance of the compensating capacitor. X_L is the inductive reactance of the tuned filter and R is its resistance.

$$X_L = \omega_o L$$

$$X_c = \frac{1}{\omega_o C}$$

$$\omega_o = 2\pi F$$

Where F is the fundamental frequency.

Let the compensating impedance be denoted by Z_{ck} *where*

$$Z_{ck} = R + j(\,kX_L - X_c/k\,) \tag{5.17}$$

let $Z_{clk} = Z_{ck}$ *in parallel with* Z_{lk}

Then

$$Z_{clk} = \frac{\{RR_{lk} - X_{lk}(kX_L - X_c/k)\} + j\{R_{lk}(kX_L - X_c/k) + RX_{lk}\}}{(R + R_{lk}) + j(X_{lk} + kX_L - X_c/k)} \tag{5.18}$$

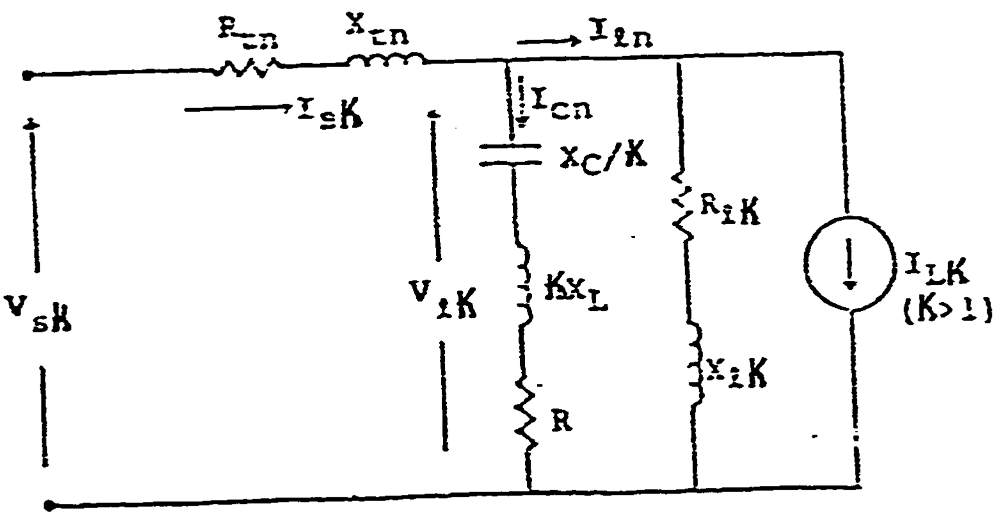


Figure 5.2 Nonlinear load with compensation .

Letting

$$R_{clk} = RR_{lk} - X_{lk}(kX_L - X_c/k)$$

and

$$X_{clk} = RX_{lk} + R_{lk}(kX_L - X_c/k)$$

Equation (5.18) can be rewritten as :

$$Z_{clk} = \frac{R_{clk} + jX_{clk}}{(R + R_{lk}) + j(X_{lk} + kX_L - X_c/k)} \tag{5.19}$$

Let also $Z_{tlk} = Z_{tk}$ *in parallel with* Z_{lk}

where

$$Z_{tlk} = \frac{(R_{tk}R_{lk} - X_{tk}X_{lk}) + (R_{tk}X_{lk} + R_{lk}X_{tk})}{(R_{tk}+R_{lk}) + j(X_{tk} + X_{lk})} \tag{5.20}$$

Letting :

$$R_{tlk} = R_{tk}R_{lk} - X_{tk}X_{lk}$$

and

$$X_{tlk} = R_{tk}X_{lk} - R_{lk}X_{tk}$$

we have

$$Z_{tlk} = \frac{R_{tlk} + jX_{tlk}}{(R_{tk}+R_{lk}) + j(X_{tk} + X_{lk})} \tag{5.21}$$

$$\begin{aligned} V_{sk} &= (R_{tk}+jX_{tk})I_{sk} + V_{lk} \\ &= (R_{tk}+jX_{tk})I_{sk} + (I_{sk}-I_{Lk})Z_{clk} \end{aligned} \tag{5.22}$$

Subsituting Z_{clk} in (5.22) , we get :

$$V_{sk} = \left(R_{tk}+jX_{tk} + \frac{R_{clk}+jX_{clk}}{(R+R_{lk}) + j(X_{lk}+kX_L-X_c/k)} \right) I_{sk}$$

$$- \frac{I_{Lk}(R_{clk}+jX_{clk})}{(R+R_{lk}) + j(X_{lk}+KX_L-X_c/k)} \tag{5.23}$$

By simplification and subsituting the expressions for R_{clk} and X_{clk} in (5.23) we get the following expression.

$$I_{sk} = \frac{V_{sk}\{(R+R_{lk})+j(X_{lk}+KX_L-X_c/k)\} + I_{Lk}(R_{clk}+jX_{clk})}{A_{rk} + jA_{in}} \tag{5.24}$$

where

$$A_{rk} = R_{tlk} + R(R_{lk}+R_{tk}) - (X_{lk}+X_{tk})(kX_L-X_c/k)$$

$$A_{ik} = X_{tlk} + R(X_{lk}+X_{tk}) - (R_{lk}+R_{tk})(kX_L-X_c/k)$$

$$I_s = \left\{\sum I_{sk}^2\right\}^{\frac{1}{2}} \tag{5.25}$$

$$V_{lk} = (I_{sk}-I_{Lk})Z_{clk} \tag{5.26}$$

$$V_{lk} = I_{sk}Z_{clk} - Z_{clk}I_{Lk}$$

substituting the expression for I_{sk} and Z_{clk} into (5.26) and simplifying ;

$$V_{lk} = \frac{V_{sk}(R_{clk}+jXclk) - I_{Lk}(\{R+j(kX_L-X_c/k)\}+(R_{tlk}+jX_{tlk}))}{A_{rk} + jA_{ik}} \tag{5.27}$$

$$V_L = \left\{\sum V_{lk}^2\right\}^{\frac{1}{2}}$$

Similarly $I_{ck} = I_{sk} - I_{Lk}$

which gives

$$I_{ck} = \frac{V_{sk}(R_{lk}+jX_{lk}) - I_{Lk}(R_{tk}+jX_{tlk})}{A_{rk} + jA_{ik}} \tag{5.28}$$

and $I_c = \left\{ I_{ck}^2 \right\}^{\frac{1}{2}}$

As before $P_L = G_L V_L^2$

and $P_s = P_L + R_T I_s^2$

Power factor

$$PF = \frac{P_L}{V_L I_s}$$

$$PF = \frac{\sum G_{lk} \sum V_{lk}^2}{\{\sum I_{sk}^2 \sum V_{lk}^2\}^{\frac{1}{2}}} \tag{5.29}$$

Efficiency (η)

$$\eta = \frac{P_L}{P_s}$$

$$\eta = \frac{\sum G_{lk} \sum V_{lk}^2}{R_T \sum I_{sk}^2 + \sum G_{lk} \sum V_{lk}^2} \tag{5.30}$$

Note that (5.24), (5.29), and (5.30) are two variable equations *in* X_L and X_c.

5.4 Resonance Condition

The reasonance condition which provides values of kX_L and X_c/k can be found by equating to zero the imaginary part of the impedance seen from the Thevenin source . These values will be eliminated from solving the two variable problem of (5.24), (5.29), and (5.30)

Let $Z = Z_{tk} + Z_{clk}$

Where Z_{clk} is given by (5.18) or (5.19) .

$$Z = R_{tk} + jX_{tk} + \frac{R_{clk} + jX_{clk}}{(R + R_{lk}) + j(X_{lk} + kX_L - X_c/k)}$$

$$= R_{lk} + jX_{tk} + \frac{\{R_{clk} + jX_{clk}\}\,\{(R + R_{lk}) - j(X_{lk} + kX_L - X_c/k)\}}{(R + R_{lk})^2 + \{X_{lk} + kX_L - X_c/k\}^2}$$

$$= R_{lk} + jX_{tk} + \frac{\{R_{clk}(R + R_{lk}) + X_{clk}(X_{lk} + kX_L - X_c/k)\}}{(R + R_{lk})^2 + \{X_{lk} + kX_L - X_c/k\}^2}$$

$$+ \frac{j\{X_{clk}(R + R_{lk}) - R_{clk}(X_{lk} + kX_L - X_c/k)\}}{(R + R_{lk})^2 + \{X_{lk} + kX_L - X_c/k\}^2}$$

Equating the imaginary part to zero :

$$X_{lk}\{(R + R_{lk})^2 + \{X_{tk} + (kX_L - X_c/k)\}^2\} + \{X_{clk}(R + R_{lk}) - R_{clk}\{X_{lk} + (kX_L - X_c/k)\}\} = 0 \tag{5.31}$$

Substituting the expressions for R_{clk} and X_{clk} in (5.31) and simplifying we get ;

$$(X_{tk} + X_{lk})(kX_L - X_c/k)^2 + \{X_{lk}^2 + 2X_{lk}X_{tk} + R_{lk}^2\}(kX_L - X_c/k) + \{R^2X_{lk} + X_{tk}\{(R + R_{lk})^2 + X_{lk}^2\} = 0 \tag{5.32}$$

Equation (5.32) can be expressed interms of a quadratic equation :

$$A(kX_L - X_c/k)^2 + B(kX_L - X_c/k) + C = 0 \tag{5.33}$$

Where

$$A = X_{tk} + X_{lk}$$

$$B = R_{lk}^2 + X_{lk}^2 + 2X_{lk}X_{tk}$$

$$C = R^2X_{lk} + X_{tk}\{(R + R_{lk})^2 + X_{lk}^2\} \tag{5.34}$$

Solving (5.33) for finding X_L and X_c we obtain :

$$kX_L - X_c/k = \frac{-A \pm \sqrt{B^2 - 4AC}}{2A} \tag{5.35}$$

By taking the solution of (5.35) where the square root of the discriminant is positive (the other solution corresponds to resonance between the load and the combination of the source impedance ie. $(R_{lk}+jX_{lk})+Z_{lk}//Z_{ci}$] . The results give us regions where X_L and X_c must not lie to prevent the occurance of resonance .

5.5 Problem Formulation

It is required here to find the optimum values of X_L^* and X_c^* to minimise the line losses $I_s^2(X_L , X_c)$ as described by (5.24) and (5.25). A necessary condition is that the differential vanish at (X_L^* , X_c^*) . *ie* .

$$\frac{\delta I_s}{\delta X_L} dX_L + \frac{\delta I_s}{\delta X_c} dX_c = 0 \qquad (5.36a)$$

Since X_L and X_c are independent. the components dX_L and dX_c are arbitrarily independent and (5.36a) implies

$$\left.\begin{aligned} \frac{\delta I_s}{\delta X_L} &= 0 \\ \frac{\delta I_s}{\delta X_c} &= 0 \end{aligned}\right\} \qquad (5.36b)$$

Unfortunately, it is not easy to solve (5.36) due to the nonlinear nature of the expression for I_s, in addition, the solution may not be unique, indicating several local maxima of the objective function . One possible approach is to apply numerical optimization of two variables which is the solution procedure used in this section.

Among the numerous optimization techniques, the minimisation of a function of N variables using the direct search polytope algorithm is chosen due to its simplicity and its availability in the form of software package for computer application . The algorithm is presented below.

5.5.1 Algorithm for using the Direct Search Polytope for minimising a function of N variables.

The Direct Search Polytope method assumes no smoothness and it is based on function comparison. It begins with n + 1 points $x_1 . x_2 . \ldots\ldots\ldots\ldots . x_{n+1}$. At each iteration. a new point is generated to replace the worst point x_j .which has the largest function value among these n + 1 points. The new point is obtained from :

$$x_k = c + \alpha(c - x_j) \qquad (5.37)$$

Where

$c = \frac{1}{n}\sum_{i \neq j} x_i$. and $\alpha(\alpha > 0)$ is the reflection coefficient.

When x_k is a best point. that is $f(x_k) \leq f(x_i)$ for i = 1. 2. .... n + 1 , an expansion point is computed ;

$x_e = c + \beta(x_k - c)$, *where* $\beta(\beta > 1)$ is called the expansion coefficient. If the new point is the worst point then the polytope would be contracted to get a better new point [16] . If the contraction step is unsuccessful. the polytope is shunk by moving the vertices half-way toward current best point. This procedure is repeated untill one of the stopping criteria is satisfied :

Criterion 1 :

$$f_{best} - f_{worst} \leq \varepsilon_f (1. + |f_{best}|)$$

Criterion 2 :

$$\sum_{i=1}^{n+1} \{f_i - \frac{\sum_{j=1}^{n+1} f_j}{n+1}\}^2 \leq \varepsilon_f$$

Where $f_i = f(x_i)$, and $f_j = f(x_j)$ are the values of the function at x_i and x_j respectively and ε_f is a given tolerance.

5.5.2 Objective function

The objective function is to minimise the line losses $I_s^2(X_L, X_C)$ discribed by equation (5.24) and (5.25) at an optimal point X_L^* and X_C^*, where these optimal values are not the solutions of the resonance equation given by (5.33) and (5.35).

minimise $I_s^2(X_L, X_C)$

for *finding* X_L^*, X_C^*

subject X_L^*, X_C^* not part of solution to (5.35)

when $I_s^2(X_L^*, X_C^*)$ is known, then the power factor and the efficiency can be calculated. Using examples one and two of chapter three and taking arbitrary values for the harmonic currents as in section 5.1. the following solutions were obtained.

Example 1 : System data

Transmission system impedance :

$$R_T = 0.02163\ \Omega,\ X_T = 0.2163\ \Omega$$

Load system impedance :

$$R_L = 1.7421\ \Omega,\ X_L = 1.6960\ \Omega$$

System harmonics = 13

Harmonic source voltage (V) :

2400.0 , 0.0 , 0.0 , 0.0 , 120.0 , 0.0 , 72.0 , 0.0 , 0.0 , 0.0 , 48.0 , 0.0 , 24.0

Frequency F = 60 Hz

Load harmonic currents (A) obtained arbitrarily :

0.0 , 0.0 , 304.4 , 0.0 , 33.0 , 0.0 , 25.0 , 0.0 , 26.0 , 0.0 , 8.0 , 0.0 , 9.0

Results

The optimal values are :

$$X_L^* = 492372.312\,\Omega$$

$$X_C^* = 492376.062\,\Omega$$

For $Q_L = 11.67\,KVAR$ and $Q_C = 11.69 KVAR$

giving us L = 1306 H and $C = 5.39\ \mu F$

minimum line current $I_s = 705.18A$

power factor $PF = 0.9937$

efficiency $\eta = 0.9936$

Example 2 : System data

Transmission system impedance :

$$R_T = 0.1\,\Omega\ ,\ X_T = 0.2\,\Omega$$

Load system impedance :

$$R_L = 1.0\,\Omega\ ,\ X_L = 3.0\,\Omega$$

System harmonics = 7

Harmonic source voltage (V) :

100.0 , 0.0 , 0.0 , 0.0 , 10.0 , 0.0 , 10.0

Frequency F = 50 Hz

Load harmonic currents (A) :

0.0 , 10.0 , 0.0 , 7.0 , 0.0 , 3.0 , 0.0

Results

The optimal values are ;

$$X_L^* = 3691.837\,\Omega$$

$$X_C^* = 3695.680\,\Omega$$

For $Q_L = 2.7087\,KVAR$ and $Q_C = 2.7059 KVAR$

giving us L = 11.75 H and C = 0.861 μF

minimum line current I_s = 10.5991A

power factor PF = 0.9195

efficiency η = 0.9885

The results of examples one and two show remarkable improvement compared with those in section 5.2 when the nonlinear load was considered without compensation . These values (ie X_L^* and X_C^*) are the values that realises minimum line current at the same time maintaining high power factor and a high transmission efficiency. In conclusion. if the load is nonlinear. the best compensation approach is the use of L - C optimal shunt compensator .

One thing still remain to be addressed that is whether these optimal values obtained from the theoretical analyses can be found among standard manufacturing values [28]. Tables 5.1 and 5.2 below are standard manufacturing data of shunt capacitors . It is clear from these that the optimal solutions for the two examples are not included in these tables . This calls for a new solution method where the manufacture's data are taken into consideration in obtaining the optimal solutions. The discrete values of the standard capacitor values are used to arrive at a solution as presented in the next section.

5.6 *Discretising The Capacitor Values*

In this section, the manufacturer's standard values for power shunt capacitors are taken into consideration. These values are considered as constraints in the sense that the solution for the capacitor should be one of the standard values . The reason for doing this is to compare the values obtained from the last section with real practical values in the market. Table 5.1 [28] shows the voltage and reactive power ratings of shunt capacitors.

Table 5.2 shows the capacitance and resistance values for McGraw - Edison capacitors with voltage ratings and reactive KVAR power ratings included. The inductive reactive values are almost continuous and there is little limitation on the manufacturer's values. Hence using only the set values for the shunt capacitors we can obtain values for the inductive reactance since equation (5.24) and and (5.25) will then become a one variable equation in X_L only. The discretising approach is described below.

Table 5.1 Voltage and Reactive Power Ratings of Shunt Capacitors

Volts, rms (Terminal-to-Terminal)	kvar	Number of Phases	BIL kV (+)
216	5, 7 1/2 and 13 1/3	1 and 3	30
240	5, 7 1/2, 10 and 15	1 and 3	30
480	10, 15, 20, 25, 35 and 50	1 and 3	30
600	10, 15, 20, 25, 35 and 50	1 and 3	30
2400	50, 100, 150 and 200	1	75
2770	50, 100, 150 and 200	1	75
4160	50, 100, 150 and 200	1	75
4800	50, 100, 150 and 200	1	75
6640	50, 100, 150 and 200	1	95
7200	50, 100, 150, 200 and 300	1	95
7620	50, 100, 150, 200 and 300	1	95
7960	50, 100, 150, 200 and 300	1	95
8320	50, 100, 150, 200 and 300	1	95
9540	50, 100, 150, 200 and 300	1	95
9960	50, 100, 150, 200 and 300	1	95
11400	50, 100, 150, 200 and 300	1	95
12470	50, 100, 150, 200 and 300	1	95
13280	50, 100, 150, 200 and 300	1	95 and 125
13800	50, 100, 150, 200 and 300	1	95 and 125
14400	50, 100, 150, 200 and 300	1	95 and 125
15125	50, 100, 150, 200 and 300	1	125
19920	150 and 200	1	125
19920	150, 200 and 300	1	125 and 150
20800	100, 150 and 200	1	150 and 200
21600	100, 150 and 200	1	150 and 200
22800	100, 150 and 200	1	150 and 200
23800	100, 150, and 200	1	150 and 200
24940	100, 150 and 200	1	150 and 200
4160 GrdY/2400	300 and 400	3	75
4800 GrdY/2770	300 and 400	3	75
7200 GrdY/4160	300 and 400	3	95
8320 GrdY/4800	300 and 400	3	95
12470 GrdY/7200	300 and 400	3	95
13200 GrdY/7620	300 and 400	3	95
13800 GrdY/7960	300 and 400	3	95
14400 GrdY/8320	300 and 400	3	95

Table 5.2 Capacitance and Resistance values for McGraw - Edison Edisol Capacitors

	100 KVAR Capacitors			150 KVAR Capacitors			200 KVAR Capacitor		
Capacitor Voltage	Normal Capacitance Range (µF)	Capacitance of a Partially Failed Unit (µF)	Approx Discharge Resistance (megohms)	Normal Capacitance Range (µF)	Capacitance of a Partially Failed Unit (µF)	Approx Discharge Resistance (megohms)	Normal Capacitance Range (µF)	Capacitance of a Partially Failed Unit (µF)	Appr Dis-char Resi tanc (meg ohm
2400	46.1–49.7	>92.1	1.4	69.1–74.6	>138	0.93	92.1–99.5	>184	0.
2770	34.6–37.3	>69.1	1.8	51.9–56.0	>104	1.2	69.1–74.7	>138	0.
4160	15.3–16.3	>23.0	3.7	23.0–24.8	> 34.5	2.5	30.7–33.1	> 46.0	1.
4800	11.5–12.4	>17.3	4.8	17.3–18.7	> 25.9	3.2	23.0–24.9	> 34.5	2.
6640	6.02–6.50	> 8.02	8.6	9.2–9.75	> 12.0	5.7	12.0–13.0	> 16.0	4.
7200	5.12–5.53	> 6.82	9.9	7.68–9.24	> 10.2	6.6	10.2–11.1	> 13.6	5.
7620	4.57–4.93	> 5.71	11.0	6.85–7.40	> 8.57	7.5	9.14–9.87	> 11.4	5.
7960	4.19–4.52	> 5.23	12.0	6.28–6.78	> 7.85	7.8	8.37–9.04	> 10.5	5.
9960	2.67–2.89	> 3.21	18.0	4.01–4.33	> 4.81	12.0	5.35–5.78	> 6.42	9.
12470	1.71–1.84	> 1.95	27.0	2.56–2.76	> 2.92	18.0	3.41–3.68	> 3.90	13.
13280	1.50–1.62	> 1.72	30.0	2.26–2.44	> 2.58	20.0	3.01–3.25	> 3.44	15.
13800	1.39–1.50	> 1.59	32.0	2.09–2.26	> 2.39	22.0	2.79–3.01	> 3.18	16.
14400	1.28–1.38	> 1.46	35.0	1.92–2.07	> 2.19	23.0	2.56–2.76	> 2.92	18.
19920	–	–	–	1.00–1.04	> 1.09	43.0	1.34–1.38	> 1.46	32.
21600	–	–	–	.853–.921	> .930	49.0	1.14–1.23	> 1.24	37.

The equation relating the reactive capacitive power Q_c in KVAR to the reactance of the capacitor is given by :

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$Q_c = \frac{V^2}{X_c} \tag{5.38}$$

$$X_c = \frac{V^2}{Q_c} \tag{5.39}$$

and from which

$$C = \frac{1}{\omega X_c} \tag{5.40}$$

5.6.1 Formulation of the Discrete Problem

Each of the values of the reactive power ratings Q_{ci} of the particular voltage . is used to calculate the corresponding value of X_c as shown in (5.39) above. This value is then substituted into (5.24) by which (5.25) now becomes a one variable equation in X_L which can be solved by using the Golden Search method as described in earlier chapters. Let $Q = \{ Q_{c1}, Q_{c2}, \ldots\ldots, Q_{cN} \}$ Then the problem can be formulated as;

Minimise $\quad I_s^2(X_L, X_{ci})$

subject $\quad X_{ci} = \dfrac{V^2}{Q_{ci}}$

$Q_{ci} \in Q$

Q_{ci} are the data in tables 5.1 and 5.2 , Q_c is the desired solution .

Algorithm for solving the Discrete Problem

Step 1 :

Choose the first value of the standard manufactured reactive power rating of capacitors in KVAR as shown in tables 5.1 and 5.2 from the set $Q_{ci} = \{ Q_{c1}, Q_{c2} .... Q_{cn} \}$ where n is the number of discrete values available for the particular voltage rating used and i has a starting value of 1 .

Step 2 :

Using only the selected value of Q_{ci}. *calculate* X_{ci} from equation (5.39)

Step 3 :

Substitute the value of X_{ci} into the objective function $I_s^2(X_L, X_{ci})$ of equation (5.24). This function now becomes a one variable problem in X_L *ie* $I_s^2(X_L)$

Step 4 :

Using the Golden Search algorithm solve for the optimal X_L.

Step 5 :

If i = n stop otherwise replace i by i = i + 1 and go to step 1 .

Step 6 :

After stopping scan through to get the global minimum.

5.6.2 Examples

The results obtained for examples 1 and 2 are :

Example 1 :

Minimum line current occured at $X_L^* = 27255.996\,\Omega$ *When* $X_c^* = 57600.0\,\Omega$

with $Q_L = 211.33\,KVAR$ and $Q_c = 100.00\,KVAR$

$I_s = 922.61\,A$

$PF = 0.7153$

$\eta = 0.9877$

For this example figure 5.3 shows a plot of the line current against the values of the capacitor (in μF) and the inductor (in Henry).

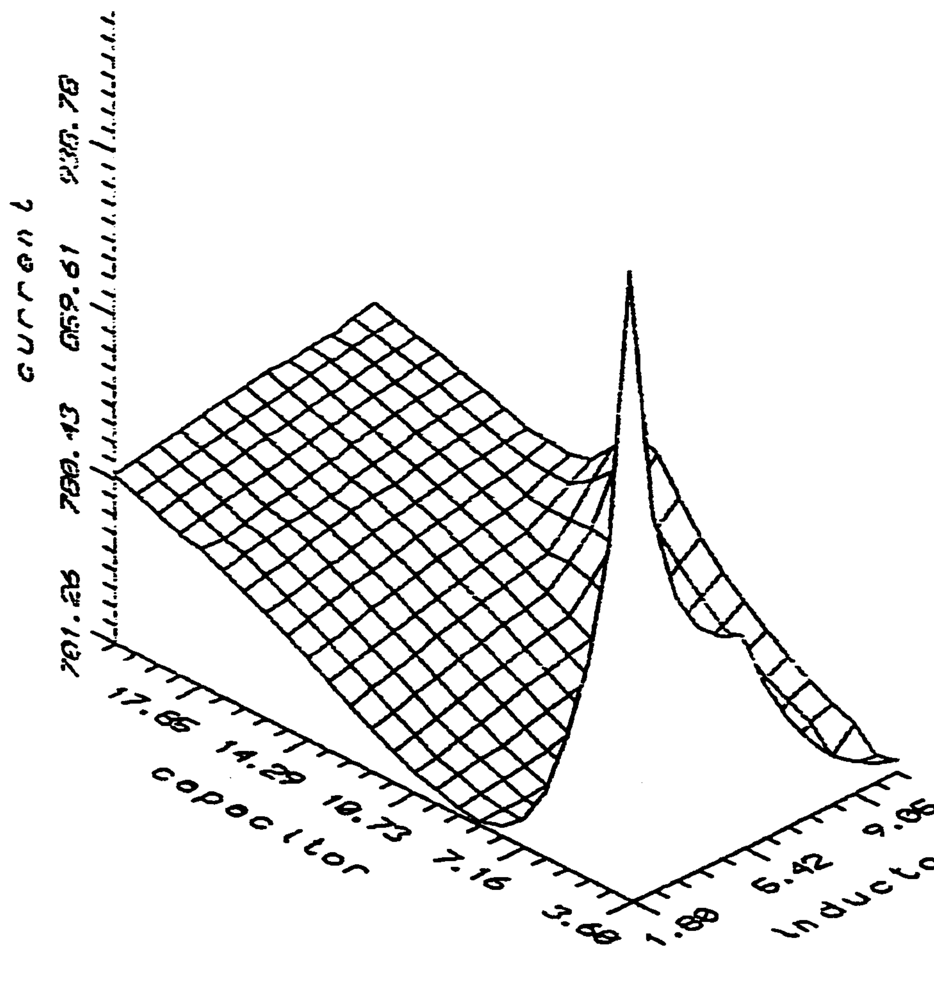


Figure 5.3 A plot of current against C and L of example 1

Example 2 :

Minimum line current occured at $X_L' = 164543.875\,\Omega$ *When* $X_c' = 1000.0\,\Omega$

with $Q_L = 0.06077\ KVAR$ and $Q_c = 10.00\ KVAR$

$I_s = 29.5631\ A$

$PF = 0.313$

$\eta = 0.909$

From these results, it can be observed that the practical values that minimises the line current has been achieved at the expense of the power factor which is reduced in this case and the line current (or losses) which increases in this case. given in table 5.3 below.

Table 5.3 Summary of results for nonlinear loads

Data	Without compensation	Theoretical solution for LC	Solution by discretising for LC	Solution for C when load is linear
Example 1 :				
	L = 0 **C = 0**	**L = 1306 H** $C=5.39\mu F$ -	**L = 72.30 H** $C=0.460\mu F$ -	**L = 0** $C=0.8764 mF$
Line current	I_s = 963.59 A -	I_s = 705.18 A -	I_s = 922.61 A -	I_s = 770.77 A
Power factor	**PF = 0.684**	**PF = 0.9937**	**PF = 0.7153**	**PF = 0.917**
efficiency	η = 98.66 % -	η = 99.36 % -	η = 98.77 % -	η = 99.25 %
Example 2 :				
	L = 0 **C = 0**	**L = 11.75 H** $C=0.861\mu F$ -	**L = 523.76 H** $C=3.183\mu F$ -	**L = 0** $C=0.181 mF$
Line current	I_s = 31.82 A -	I_s = 10.59 A -	I_s = 29.56 A -	I_s = 25.91 A
Power factor	**PF = 0.294**	**PF = 0.9195**	**PF = 0.3133**	**PF = 0.354**
efficiency	η = 89.62 % -	η = 98.85 % -	η = 90.90 % -	η = 93.01 %

CHAPTER SIX

COMMENTS AND CONCLUSIONS

6.0 COMMENTS

6.1 Definition of Power Terms

The definitions of power terms as seen in chapter two have general concensus when linear sinusoidal circuits are considered. However, the discussion of the reactive power concept has received much renewed attention because of the distortion in the power system currents and voltages due to the increasing use of static converters. Budeanu was the first to propose a definition for the reactive power for periodic waveforms expressed as :

$$Q = \sum V_k I_k \sin\theta_k \tag{6.1}$$

Budeanu's reactive power definition has been shown to be deficient in the sense that one fundamental aspect was not taken into account . This aspect is that ; negative and positive reactive powers do compensate if they correspond to the same frequency, but this is not the case if they correspond to different frequencies. To make this point clear, consider a capacitor and an inductor connected in parallel to a sinusoidal voltage source v . It was demonstrated in chapter two that oscillating power transfer at twice the supply frequency is needed to realise the fluctuating energy $\frac{CV^2}{2}$ stored in the capacitor C and $\frac{Li^2}{2}$ stored in the inductor L (these equations were shown in the maxwell's equations in (2.19)) . Since the inductor and the capacitor currents are in quadrature with respect to the voltage , but the capacitor current is leading and the inductor current is lagging. the energy in the capacitor increases when the the energy in the inductor is decreasing . and vice - versa . The voltage source therefore has to supply only the *difference* of the oscillating power components and this fact was shown in Emanuel- 's paper [18] . Consider now a nonsinusoidal source with a voltage consisting of a fundamental frequency component and a third harmonic . It is connected to a reactive element which has a negative or capacitive reactance at the fundamental frequency and a positive or inductive reactance at the third harmonic frequency. The fluctuations of the stored capacitive and inductive energies are not synchronous in this case. such that the pulsating power to be delivered by the source does not correspond to the difference of both energy components. Therefore there is no justification for simply adding the reactive power corresponding to different frequencies . as is done in Budeanu's reactive power concept. Moreover. there are also cross-terms in the stored energy expression corresponding to the product of the fundamental current and the third harmonic current in the expression of the inductive energy.

For a generalised definition of the reactive power concept, the following questions should be considered ;

(i) Which features make the concept of reactive power in sinusoidal situations so intresting for the analysis of practical power system operation ? .

(ii) How can the concept of reactive power be generalised to nonsinusoidal situations in such a way that these properties or at least some of them are retained ? .

With respect to question (i) the following interesting situations may be stated :

(a) The reactive power is the amplitude of the fluctuating component of the supplied power which can be compensated by a parallel reactive element.

(b) The reactive power corresponds to the apparent power of the compensating reactive element which realises unity power factor .

(c) Zero reactive power is equivalent to unity power factor. It correspond to the smallest line current supplying the same active power.

(d) For an inductive line the voltage drop in the line is (approximately) proportional with the transmitted reactive power.

6.2 Linear sinusoidal circuits

The work done on linear sinusoidal circuits is the well documented and generally accepted concept of these circuits. Most a.c. load programmes that are run on the computer for the purpose of designing either a transmission or distribution network usally assume that the network is a linear sinusoidal circuit.

For the case of zero source impedance. the same value of capacitor design satisfied the criteria of both maximum power factor and minimum line losses ie

$$C_o = \frac{B_L}{2\pi F} , \qquad (6.2)$$

The third criteria - the maximum transmission line effeciency is not considered here since the source impedance is zero.

When the source impedance is taken into consideration, maximising the power factor and minimising the line losses will lead to the same capacitor design value again. This value is :

$$C_o = \frac{B_L}{2\pi F}$$

This equation is the same as equation (6.2) and it is the same equation which realises maximum line effeciency . The conclusion reached at this point is that; in a linear sinusoidal circuit whether or not the source impedance is taken into consideration, the same value of optimal shunt design capacitor will satisfy all the three criteria vis-a -vis : maximum power factor . mimimum line losses . and maximum transmission line effeciency . In a nutshell the three criteria are the same .

6.3 Linear Nonsinusoidal Circuits

A. Zero source impedance

When zero source impedace is assumed in the presence of a linear nonsinusoidal circuit. then compensation is achieved by using the two criteria: maximum power factor and minimum line losses . Both criteria led to the same optimal capacitor design given by equation (4.40) which is:

$$C_o = \frac{\sum k B_{Lk} V_{Lk}^2}{\omega_o \sum k^2 V_{Lk}^2} \tag{6.3}$$

It is clear that (6.3) is different from (6.2) and it can be concluded that in the presence of source distortion if even the source impedance is neglected the optimal shunt capacitor design is defferent from the designed value under linear sinusoidal circuits.

B. Non zero source impedance

Under this section it was clearly demonstrated that the three criteria each led to a different optimal shunt capacitor design by using a nonlinear model for the system. The advantages of the method over conventional approaches as mentioned above were shown in the results of the simutaled system data of examples 1 and 2 as tabulated in tables 4.1, 4.2 , and 4.3 . These advantages included the improvement in the accuracy of the solution and in the ability of the developed algorithm to guarantee convergence to the optimal solution. Using this method, the global optimal solution as well as the local optimum are determined. These additional information can be useful for performing a cost-benefit decision analysis before implementing the optimal capacitive compensation.

Additional comments to be added to the concept of power factor correction in nonsinusoidal systems are:

1. The linear passive capacitor value which would produce a unity power factor is not physically realisable.

2. Neglecting the line impedance in the analyses and the resonant phenomena would lead to erroneous results .

3. Due to resonance conditions. an increase of shunt capacitive compensation does not necessarily produce an improved power factor operation as predicted by the fundamental frequency analyses. This conclusion is verified by the results of example two in chapter four.

4. Due to the uncertainties in system parameter data , one might consider a suboptimal capacitive compensation if resonant conditions occur at various capacitor values very close to the optimal value.

5. The harmonic voltage distortion generated by a nearby customer could produce to another customer. a low power factor operating problem of which a simple and economic solution may not be feasible.

The final comment is that, whether or not the solution generated by this method is indeed optimal depends on the knowledge of the system configuration, operating condition , and harmonic voltage distortion. This fact was shown in the case of example 2 which never produced any substantial increase in power factor under the penalty function discussion because of the limitation on the practical values of capacitors. Nevertheless, the developed method represent a useful tool for providing an optimal solution under a given situation.

6.4 Non Linear Nonsinusoidal Circuits

Under this section the best compensational approach was the use of shunt inductor-capacitor (LC) compensators if the compensator is to provide optimum average power factor without being subjected to overload. Compared with pure capacitive compensation. LC compensation may not provide the same power factor. However. the theoretical analyses with the two examples simulated in this work. indicates that. the maximum possible average power factor attainable by LC compensators is substantially higher than that obtainable by pure capacitive compensators. Note that for power factors higher than those attainable by pure capacitive compensation . the LC combination is then essentially functioning simultaneously as a compensator as well as a filter. This benefit can be made use of if and only if the LC solution lies within the practical standard manufacturer's values of capacitors. When the LC solution producing the optimal capacitor value does not lie within manufucturing standards, then it is preferrable to use these standard values to solve for the optimal conditions. In such a situation there is bound to be difference in the optimal solution from that obtained theoretically. Depending on the system configuration . the average maximum power factor may be less than before. In view of the fact that LC act both as a compensator and a filter , it is worth trading off the higher power factor attainable by pure capacitive compensation for the lower power factor in the presence of nonlinear loads. In this case the LC will filter out the load harmonic currents thereby preventing these harmonics from being fed into the network to affect other linear load busbars.

6.5 Conclusion

It can be concluded from this work that eraneous results are obtained in assuming that the source is devoid of harmonics or that the source impedance are negligible. From the theoretical analysis and the results obtained for the numerical examples, it can be deduced that the approach of this work has provided a major improvement over the conventional approaches by guaranteeing convergency to the optimal solution. The following observations can be derived :

1. The three criteria. maximisation of the transmission effeciency. minimisation of the line currents. and maximisation of the power factor. leads to different optimal capacitor values.

2. Chapter four of this work is to applicable linear loads even if the structure is more complex than the two examples used. This is confirmed by the fact that varying the harmonics values by ± 10 *per cent* to include the sensitivity factor produced thesame optimal capacitor values.

3. The penalty function approach is an effective tool for designing a shunt compensator to meet all the three criteria using only one objective function. This approach can reveal system designs which will lead to very low power factors which can not be improved beyond some practical value.

4. For a nonlinear load. it is necessary to use LC compensators. Such compensators have dual purpose. The first is that. it acts as a compensator to improve the power factor of the nonlinear load. Secondly, it acts as a filter of the harmonic load currents thus preventing the proliferation of the network with these currents.

5. For some particular network parameters. the design of an LC compensator might not be a practical value since shunt capacitors are manufactured in set values of KVAR ratings. Under such condition the disceret apprach is necessary to provide a practical solution.

6.6 Future Work

A future extension of this work is neccessary to include a cost benefit analysis to obtain the most economic shunt compensator. This can be done when the cost of the compensator. the cost of energy losses. the time variation of the load and the source impedance are taken into consideration. Since the power factor is a multimodal function. a much cheaper compensator can in some cases be used at the expense of very little performance deterioration.

An important aspect is to develop a practical procedure by means of which the source harmonics and the load harmonics of a system can be determined accurately. This officource should include the determination of the sensitivity of the system.

APPENDICES

APPENDIX A :

A.1 Physical Interpretation of Active . Reactive and Apparent Powers in a Sinusoidal System

To demonstrate the physical interpretation of the active. reactive . and apparent powers in a sinusoidal system. the practical circuit of figure A.1 is used for the illustration.

A.2.1 Physical Nature of the Active Power

The instantaneous load voltage ' e ' and cuurent ' i ' are given by :

$$e = iR + L\frac{di}{dt} \qquad \text{(A.1)}$$

$$= E_m sin(\omega t)$$

$$i = \frac{E_m}{|Z|} sin(\omega t - \Phi) \quad = I_m \sin(\omega t - \Phi) \qquad \text{(A.2)}$$

Where $|Z| = \sqrt{R^2 + \omega^2 L^2}$ (A.3)

and $\Phi = \tan^{-1}(\frac{\omega L}{R})$ (A.4)

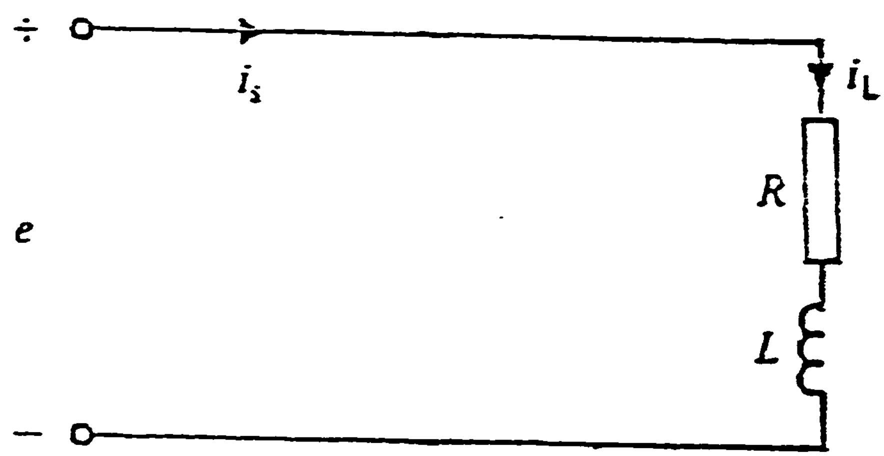


Fig. A.1 Series R - L circuit for illustration.

also $e_R = iR = \dfrac{E_m R}{|Z|} \sin(\omega t - \Phi)$ (A.5)

$$e_L = L\frac{di}{dt} = \frac{E_m}{|Z|}\omega L\sin(\omega t - \Phi) \quad \text{(A.6)}$$

By KVL

$$e = e_R + e_L \quad \text{(A.7)}$$

The instantaneous power p is given by :

$$p = ei \quad \text{(A.8a)}$$

$$= \frac{E^2}{|Z|}\cos\Phi(1-\cos 2\omega t) - \frac{E^2}{|Z|}\sin\Phi \sin 2\omega t \quad \text{(A.8b)}$$

$$= EI\cos\Phi(1-\cos 2\omega t) + EI\sin\Phi \sin 2\omega t \quad \text{(A.8c)}$$

$$= \frac{E^2}{|Z|}\{\cos\Phi - \cos(2\omega t - \Phi)\} \quad \text{(A.8d)}$$

Where

$$E = \frac{E_m}{\sqrt{2}} \text{ and } I = \frac{I_m}{\sqrt{2}} \quad \text{(A.9)}$$

E and I are the effective (rms) values of e and i respectively.

NB: We can rewrite equation (A.8c) in the form ;

$$p = p_p + p_q \quad \text{(A.10)}$$

Where $p_p = P(1 - \cos 2\omega t)$ (A.11a)

p_p is the instantaneous active power . P is the average (real) active power given by

$$P = EI\cos\Phi \quad \text{(A.11b)}$$

Similarly the instantaneous reactive power is given by ;

$$p_q = Q\sin 2\omega t \quad \text{(A.12a)}$$

Q is the maximum value of the pulsating reactive power defined from (A.8c) as:

$$Q = EI\sin\Phi \qquad (A.12b)$$

The component p_r has the average value P. and it is a unidirectional pulsating power fluctuating between 0 and 2P. This fluctuation is inherent to an alternating voltage source and the term p_r is called the intrinsical instantaneous power.

The instantaneous power p in (A.8) may be thought of as consisting of two analytical components - the real (or average) power P of (A.11b) plus an oscillating component of double frequency as can be seen in equation (A.8d). P is seen as the D.C. component of p and therefore the physical nature of P, the active power . may be defined as the constant power which is taken from the A.C. system, in the case of a load. or which is supplied to the A.C. system . in the case of a source. Time variation of $p = ei$ is given in figure A.2 . Positive values of ei represent power trasferred from the supply to the load while negative values of ei represent instantaneous values of power being transferred from the magnetic feild of the inductor back into the supply.[5]

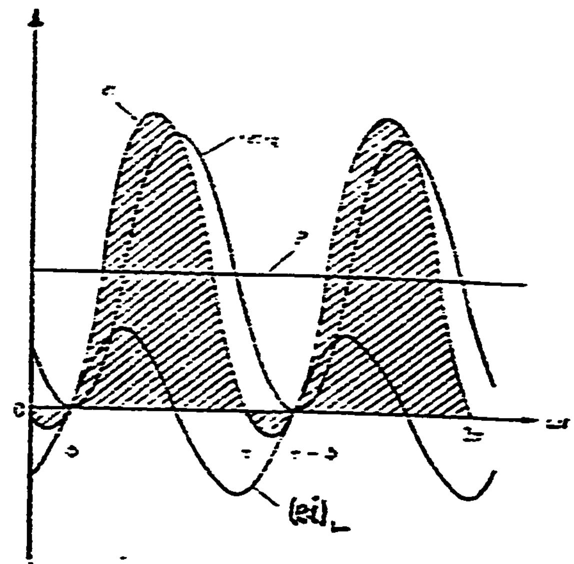


Fig A.2 Instantaneous power in sinusoidal situation

A.2.2 Physical nature of the Reactive Power

From Maxwell's equations [5]:

Kinetic energy (K.E.) is $W_T = \frac{1}{2}Li^2$ (A.13a)

Potential energy (P.E.) is $W_p = \frac{1}{2}CE^2$ (A.13b)

and the Dissipation energy is $W_c = \frac{1}{2}Ri^2$ (A.13c)

P is associated with the dissipation function W_c [5]. and the reactive power Q with the K.E. W_T and / or the P.E. W_p. Therefore P and Q should be analysed independently. If P is the D.C. component of p as stated before, then Q must be associated with the oscillating component of double frequency in (A.8c). It should be noted that the magnitude of the oscillating component is not represented by the second term in (A.8d) but found by integration over half period of the double frequency $\frac{\pi}{2}$ with a constant lower limit, which may be chosen as $0 = 0$. Hence, the average value of the sinusoidal component of (2.15) which is the reactive power is given by

$$p_q = \frac{2E^2}{|Z|\pi}\int_0^{\frac{\pi}{2}} \cos(2\omega t - \Phi)d\omega t$$

$$= \frac{2EI}{\pi}\int_0^{\frac{\pi}{2}} Cos(2\omega t - \Phi)d\omega t$$

$$= \frac{2}{\pi}EI\sin\Phi$$

$$= \frac{2}{\pi}Q \qquad \text{(A.14)}$$

From (A.14) a conclusion can be drawn that, Q must be the amplitude of the sinusoidal (oscillating) changing power. Thus. it is possible to define the physical nature of Q as the amplitude of sinusoidal power which is taken from and returned to the A.C. system during one period of power variation[5]. Reference 6 arrived at this conclusion by viewing the reactive power in terms of energy illustrated by using a lossless synchronous generator supplying a purely linear inductive or reactive load. This conclusion contradicts the idea that reative power has no physical nature as said in reference 4. Appendix B give the expressions for different circuits under different operational conditions.

A.2.3 *Physical Nature of the Apparent Power* (S_s)

The apparent power is expressed as:

$$S_s = EI$$

$$= P + jQ \qquad \text{(A.15a)}$$

Where $$S^2 = \{P^2 + Q^2\}^{\frac{1}{2}} \qquad \text{(A.15b)}$$

Knowing the physical natures of P and Q in relation to the instantaneous power p, the apparent power S_s is a figure of merit representing the maximum effective power which may be transferred from or to the system.

Equation (A.15) is obtained for the sinusoidal system . that is , when voltage and current have the same frequency. However, the physical nature of S_s is the same for a nonsinusoidal system with differences in the component of S_s.

Appendix C contains the generally accepted formulae for the above defined qualities in both sinusoidal and nonsinusoidal conditions.

APPENDIX B :

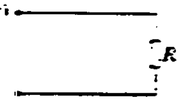
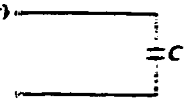
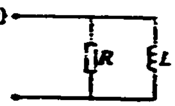
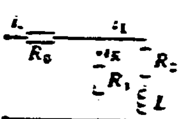
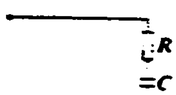
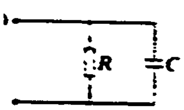
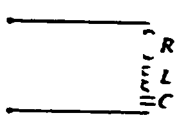
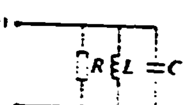
Generaly Accepted Formulae

QUANTITY	SINUSOIDAL	NONSINUSOIDAL
A1. Instantaneous	$e = E_m \sin\theta$	$e = \sum E_{km} \sin(k\theta + \psi_k)$ **k is voltage harmonic number $k = 0, 1, 2, \ldots$**
A2. Instantaneous current	$i = I_m \sin(\theta + \varphi)$	$i = \sum I_{nm} \sin(n\theta + \Delta_n)$ **where n = 0, 1, 2...**
A3. Instantaneous power	$p = ei$ - $= (E_m \sin\theta x I_m \sin(\theta + \varphi)$ $= EI\cos\varphi(1 - \cos 2\omega t)$ $EI\cos\varphi(1 - \cos 2\omega t)$ **= P (1 - cos 2 omega t) +** $Q\sin 2\omega t$	$p = ei$ $= (\sum E_{km} \sin(k\theta + \psi_k))$ $x(\sum I_{nm} \sin(n\theta + \Delta_n)$
A4. RMS voltage	$E = \sqrt{\frac{1}{T}\int_0^T e^2 dt}$ $E = \frac{E_m}{\sqrt{2}}$	$E = \sqrt{\frac{1}{2}\sum E_{km}^2}$ $= \sqrt{\sum E_k^2}$
A5. RMS current	$I = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$ $I = \frac{I_m}{\sqrt{2}}$	$I = \sqrt{\frac{1}{2}\sum I_{nm}^2}$ $= \sqrt{\sum I_n^2}$
A6. Apparent power	$S = EI$	$S = EI$

	$= P + jQ$ -	$S^2 = P^2 + Q^2 + D_r^2$
	$S^2 = P^2 + Q^2$ -	$S^2 = E^2 I^2$
		$= \sum_{k=0}^{\infty} E_k^2 \sum_{n=0}^{\infty} I_n^2$
		$= \sum_{k=0}^{\infty} E_k^2 I_k^2 +$
		$\sum_{k=0, n=0, k \neq n}^{\infty} E_k^2 I_n^2$
A7. Active power	$P = \frac{1}{T}\int_0^T p\,dt$ -	$P = \sum E_k I_k \cos\varphi_k$
(Average power)	$P = EI\cos\varphi$ -	
A8. Reactive power	$Q = EI\sin\varphi$ -	$Q = \sum E_k I_k \sin\varphi k$
A9. Distortion power		$D_r = S^2 - P^2 - Q^2$
		$= \sum_{k=0, n=0, k \neq n}^{\infty} E_k^2 I_n^2$

APPENDIX C :

Instantaneous voltampere components.

Circuit	ei	$(ei)_R$	$(ei)_{L \text{ or } C}$
(a) 	$\frac{E^2}{R}[1-\cos 2\omega t]$	$\frac{E^2}{R}[1-\cos 2\omega t]$	
(b) 	$\frac{E^2}{\omega L}\sin 2\omega t$		$-\frac{E^2}{\omega L}\sin 2\omega t$
(c) 	$E^2\omega C \sin 2\omega t$		$E^2\omega C \sin 2\omega t$
(d) 	$\frac{E^2}{\lvert Z\rvert}\cos\Phi(1-\cos 2\omega t)$ $\frac{E^2}{\lvert Z\rvert}\sin\Phi\sin 2\omega t$	$\frac{E^2}{\lvert Z\rvert}\cos\Phi[1 -\cos 2(\omega t-\Phi)]$	$\frac{E^2}{\lvert Z\rvert}\sin\Phi\sin 2(\omega t-\Phi)$
(e) 	$\frac{E^2}{\lvert Z\rvert}\cos\Phi(1-\cos \omega t)$ $\frac{E^2}{\lvert Z\rvert}\sin\Phi\sin 2\omega t$	$\frac{E^2}{\lvert Z\rvert}\cos\Phi(1 -\cos 2\omega t)$	$-\frac{E^2}{\lvert Z\rvert}\sin\Phi\sin 2\omega t$
(f) 	$\frac{E^2}{Z}\cos\Phi(1-\cos \omega t)$ $\frac{-E^2}{Z}\sin\Phi\sin 2\omega t$	$(ei)_{R_s} = i_s^2 R_s$ $(ei)_{R_1} = \frac{e_1^2}{R_1}$ $(ei)_{R_2} = i_L^2 R_2$	$(ei)_L = e_L i_L$
(g) 	$\frac{E^2}{Z}\cos\Phi(1-\cos 2\omega t)$ $\frac{E^2}{Z}\sin\Phi\sin 2\omega t$	$\frac{E^2}{\lvert Z\rvert}\cos\Phi[1 -\cos 2(\omega t-\Phi)]$	$\frac{E^2}{\lvert Z\rvert}\sin\Phi\sin 2(\omega t-\Phi)$
(h) 	$\frac{E^2}{\lvert Z\rvert}\cos\Phi(1-\cos 2\omega t)$ $\frac{E^2}{\lvert Z\rvert}\sin\Phi\sin 2\omega t$	$\frac{E^2}{\lvert Z\rvert}\cos\Phi(1 -\cos 2\omega t)$	$\frac{E^2}{\lvert Z\rvert}\sin\Phi\sin 2\omega t$
(i) 	$\frac{E^2}{Z}\cos\Phi(1-\cos 2\omega t)$ $-\frac{E^2}{Z}\sin\Phi\sin 2\omega t$	$\frac{E^2}{Z}\cos\Phi[1 -\cos 2(\omega t-\Phi)]$	$(ei)_L = \frac{E^2\omega L}{Z}\sin 2(\omega t -\Phi)$ $(ei)_C = -\frac{E^2}{\omega C Z}\sin 2(\omega t -\Phi)$
(j) 	$\frac{E^2}{\lvert Z\rvert}\cos\Phi(1-\cos 2\omega t)$ $-\frac{E^2}{Z}\sin\Phi\sin 2\omega t$	$\frac{E^2}{\lvert Z\rvert}\cos\Phi(1 -\cos 2\omega t)$	$(ei)_L = -\frac{E^2}{\omega L}\sin 2\omega t$ $(ei)_C = E^2\omega C\sin 2\omega t$

NOMENCLATURE

R_T , X_T = Fundamental values of the resistance and reactance of the transmission source.

R_{kT} . X_{kT} = Harmonic values of the resistance and reactance of the transmission source.

R_L . X_L = Fundamental values of the resistance and reactance of the load

R_{kL} , X_{kL} = Harmonic values of the resistance and reactance of the load

G_L , B_L = Fundamental values of the conductance and susceptance of the load

G_{kL} , B_{kL} = Harmonic values of the conductance and susceptance of the load

I_{kL} = Injected harmonic load currents from the nonlinear load

I_{ks} = *source harmonic currents*

$I_s = \sum I_{sk}$ *source current*

V_{ks} = *source harmonic Voltages*

$V_s = \sum V_{sk}$ *source voltage*

I_{kL} = *load harmonic currents*

V_{kL} = *load harmonic Voltages*

$V_L = \sum V_{Lk}$ *load voltage*

$$\Phi_k = \psi_i - \Delta_k$$

REFERENCES

1. Gill G. Richards et al. " Optimal LC compensators for nonlinear loads ", IEEE / PES 1988 Winter Meeting.

2. G. T. Heydt . " Electrical power quality " , Stars in a circle publications . 1991.

3. W. Sphepherd and P. Zand . " Energy flow and power factor in nonsinusoidal circuits ", Cambridge university press , 1979.

4. Chu F. Ronnie and R.H. Avendano. " A Direct Method for Identify ing the Optimal Power Factor Correction in Nonsinusoidal Systems" .IEEE Transaction on Power Apparatus and Systems.vol.PAS-104, No.4, April 1985

5. J. L. Willems. " Power factor correction for distorted bus voltages". To be published in Electric machines and power systems.

6. Czarnecki.S. Leszek. " Minimisation of Distortion Power of Nonsinusoidal sources applied to Linear loads ".IEE PROC. vol.128, Pt.c, No.4, July 1981,pp.208 - 210

7. Raonic.Dusan.D. Cyganski,and A.E. Emanuel." Power Factor Compensation At Busses with Slightly Distorted Voltage due to Random Harmonics",IEEE / PES ,1988.pp.1 - 6

8. Leszek S. Czarnecki . " Considerations on the reactive power in nonsinusoidal situations", IEEE Transactions on instrumentation and measurement, Vol. IM-34, No. 3, Sept. 1985 9. Leszek S. Czarnecki . " Minimisation of distortion power of nonsinusoidal source applied to linear loads ", IEE Proceedings, Vol. 128, Pt. C. No. 4, July 1981.

10. P. S. Filipski. " Polyphase apparent power and power factor under distorted waveform conditions ". IEEE Transaction . Vol.6. No. 3. July 1991

11. J.J. Grainger. and S.H. Lee."optimum Size and Location of Shunt Capacitor for Reduction of Losses on Distribution Feeders".IEEE Transact ion on Power Apparatus and Systems ".vol.100.March 1981.pp.1105 - 1118

12. Arrillaga.J.D. Bradely.P.S. Bodger ." Power systems Harmonics". John Wiley & sons.London.1989

13. Baghzouz.Y. and S. Ertem. " Shunt Capacitor Sizing for Radial Distribution Feeders with Distorted Substation Voltages ".IEEE Transaction on Power Delivery. vol.5. No.2. April 1990. pp.650-657 pp.959-964

14. Chiang.Hsiao-Dong.et al. " Optimal Capacitor Placement in Ditribution systems:A New Formulation and the Overall Problem " .IEEE transaction on Power Delivery ". vol.5. No.2. April 1990. pp.634-642

15. Chiang.Hsiao-Dong,et al. " Optimal Capacitor Placement in Ditribution systems:Part 2 : Solution Algorithms and Numerical Results" ,IEEE transaction on Power Delivery ", vol.5, No.2, April 1990, pp.643-649

16. M. A. Slonim and J. D. Van Wyk . " Power components in a system with sinusoidal and nonsinusoidal voltages and/or current ", IEE Proceedings. Vol.135, Pt.B, No. 2, March 1988.

17. Alexander Eigeles Emanuel et al. " Power in nonsinusoidal situations - a review of definitions and physical meaning ", IEEE Transactions on power delivery , Vol.5, No. 3, July 1990.

18. M. T. Chen et al. " Power component definitions and measurements *for a harmonic-polluted power circuits ". IEE Proceedings-C. Vol.138, No. 4, July 1991.*

19. N. L. Kusters and W. J. M. Moore. " On the definition of reactive power under nonsinusoidal conditions ". IEEE Transaction on power apparatus and systems . Vol. PAS-99. NO. 5. Sept./Oct. 1980

20. N. L. Kuster and W. J. M. Moore. " Additional discussion to ' Reactive power under nonsinusoidal condition' ". IEEE Transactions Vol. PAS-102, NO. 4. April 1983.

21. Willems, J.L. " Distortion of the Definition of Reactive Power Under Nonsinusoidal Conditions ".IEEE Transactions on Power Apparatus and Systems.vol.PAS-102.1983.pp.1023 - 1024

22. Suat Ertem and Yahia Baghzouz . " Optimal shunt capacitor sizing for distribution systems with multiple nonlinear loads " . Industrial and commercial power systems technical conference 1990

23. G. G. Richards and O.T. Tan." Comments on 'Consideration on th Reactive Power Power in Nonsinusoidal Situations' ".IEEE Trans. Istrum. Measur. vol.IM-35, Sept.1986, pp.365 - 366.

24. Baghzouz,Y. " Effects of Nonlinear Loads on Optimal Capacitor Placement in Radial Feeders",IEEE Transactions on Power Delivery, vol.6, January 1991,pp.245-251

25. Ertem. Suat and Y. Baghzouz. "Optimal Shunt Capacitor Sizing for Distribution Systems with Multiple Nonlinear Loads",Industrial and Commercial Power Systems Technical Conference. 1990.pp.51 - 56

26. Gill G. Richards and Owen T. Tan. " Correspondence- comments on *considerations on the reactive power in nonsinusoidal situations" IEEE Transactions on instrumentation and measurement. Vol.IM-35. NO. 3. Sept. 1986*

27 Mokhtar S. Bazaraa and C. M. Shetty. " Nonlinear programming" John Wiley and sons. New York 1979.

28 . " Multivariate function ", IMSL. Inc. Math Libray

29. " IEEE standard for shunt power capacitors " IEEE. December 15. 1980.

30. Emanuel A. E. " Energetical factors in power systems with nonlinear load ", Archiv for Elektrotechnik, 1977.

VITA

Bawah. A. Umar received the Bachalor of Science degree in Electrical Engineering from the University of Science and Technology (UST). Kumasi . Ghana in 1987. From 1987 to 1989 he worked with the Northern Electricity Department of the Volta River Authority of Ghana as an Electrical engineer on National service.In September 1989 , he joined the King Fahd University of Pertroleum and Minerals (KFUPM). Saudi Arabia. on an employment as a research assistant in the Electrical Engineering Department. While working as a research assistant he also enrolled for the M.S. Electrical engineering in thesame department and graduated in 1992 .